ME 3351 ENGINEERING MECHANICS

UNIT - 1

STATICS OF PARTICLES

Syllabus :-

Fundamental Concepts and Principles, System of units, Method of Problem solutions, Statics of Particles - Forces in a plane, Resultant of Forces, Resolution of a Force into Components, Rectangular Components of a Force, unit Vectors. Equilibrium of Particle - Newtons first Law of Motion, Space and Free-body diagrams, Forces in space, Equilibrium of Particle in space.

Fundamental Concepts and Principles :

Mechanics - is the physical science that deals with the behaviour of bodies that are acted upon by forces.

Statics - is the study which deals with the conditions of bodies in equilibrium subjected to external forces.

Equilibrium - When the force acting on the body is balanced the system has no external effects on the body, the body is in equilibrium.

Dynamics :- It is also a branch of mechanics in which the forces and their effects on the bodies in motion are studied.

Dynamics is subdivided into two parts
(i) kinematics
(ii) Kinetics

Kinamatics deals with the geometry of motion of bodies without and application of external forces.

Kinetics deals with the motion of bodies with the application of external forces.

Hydromechanics :-

It is the study of conditions of fluid under which it can remain at rest or in motion. Hydro~~dynamics~~mechanics can be divided into two parts, Hydro statics & hydrodynamics.

Hydro statics-

It is the study of fluid at rest.

Hydro dynamics :-

It is the study of fluid under motion.

Rigid Body :-

A body is said to be rigid if it retain its shape and size even if the external forces are applied on it. It is called rigid body.

Some basic Terms used in Mechanics:-

Mass :- The quantity of matter possessed by a bobdy is called mass. The mass of the body can not change unless the body is damaged and part of it is physically seperated.

Length :- It is a concept to measure linear distances.

Time :- Time is the measure of succession of events. The successive event selected is the rotation of earth about it own axis and this is called a day.

Space :- Any geometric region in which the study of body has been done is called space.

Displacement :- It is defined as the distance moved by a body / particle in a specified direction.

Velocity :- The rate of change of displacement with respect to time is defined as Velocity.

Acceleration :- It is the rate of change of velocity with respect to time.

Momentum :- The product of mass and velocity is called momentum. Thus

Momentum = Mass × Velocity.

LAWS OF MECHANICS :-

The followings are the fundamental laws of mechanics

(i) Newton's first law
(ii) Newton's Second law
(iii) Newton's third law
(iv) Newton's law of gravitation
(v) Law of transmissibility of forces
(vi) Parallelogram law of forces

Newton's first law :

It states that, every body continues in its state of rest or of uniform motion in a straight line unless it is compelled by an external agency acting on it.

Newtons Second law :-

It states that the rate of change of momentum of a body is directly proportional to the impressed force and it takes place in the direction of force acting on it.

According to this law,

Force = Rate of change of momentum

But momentum = mass × velocity

As mass donot change ⇒ Force = mass × rate of change of velocity

∴ F = mass × acceleration

$$\boxed{F = m \times a}$$

Newton's third law :-

It states that for every action, there is an equal and opposite reaction.

UNITS AND DIMENSIONS OF QUANTITES.

Units :-

Measurements are always made in comparison with certain standards. For(eg) — 2.5 m long cloth piece.

Here 'm' – meter is the unit of length

There are four systems of units are used for the measurement of physical quantities.

(i) FPS System – (Foot - Pound - Second)

(ii) CGS System – (Centimetre - Gram - Second)

(iii) MKS System – (Meter - Kilogram - Second)

(iv) SI System – (System of international units)

SI Units (International System of units) :-

The fundamental units of the system are metre (m) for length, Kilogram (Kg) for mass and Second (S) for time.

The unit of force is newton (N). One newton is the amount of force required to induce an acceleration of $1 \, m/sec^2$ on one Kg of mass.

Dimensions :-

The branch of mathematics dealing with dimensions of quantities is called dimensional anlysis.

(i) Absolute system (MLT system)

(ii) Gravitational system (FLT system)

Absolute system :- (MLT system)

* A system of units defined on the basis of length, time and mass is referred to as an absolute system.

* According to SI system of units, three basic units metre, second and kilogram can be used. In MLT system M - refers to mass, L - refers to Length and T - refers to Time.

Gravitational system :- (FLT system)

* Systeml of units defined on the basis of length, time and force is referred to as a gravitational system

* In this system, force is measured in a gravitational field. Thus its magnitude depends upon the location where the measurement is made. FLT system refers to the Force - Length - Time system.

Basic units :- The units which are used for measurement of basic or fundamental quantites (mass, length and time) are known as basic units or fundamental units.

Eg. Length, mass and time

2. Derived units :-

All the units which are used for the measurement of physical quantities other than fundamental units are known as derived units or secondary units.

(eg) Velocity, Acceleration, force, density etc.

Sl. No	Quantity	Units	Dimensions in MLT system
1.	Length	m	L
2.	Area	m^2	L^2
3.	Volume	m^3	L^3
4.	Density	Kg/m^3	ML^{-3}
5.	Mass	Kg	M
6.	velocity	m/sec	LT^{-1}
7	Acceleration	m/sec^2	LT^{-2}
8.	Force	Newton (N)	MLT^{-2}
9.	Discharge	m^3/sec	$\cancel{M}\ L^3T^{-1}$
10.	Power	Nm/sec (watt)	ML^2T^{-3}
11.	Area moment of inertia	m^4	L^4
12	Mass moment of inertia	kg/m^2	ML^{-2}
13.	Modulus of Elasticity	N/m^2	$ML^{-1}T^{-2}$

$1\ N = 10^5$ Dyne

$1\ Kg F = 9.81\ N.$

VECTORS :-

Vector quantities used in engineering mechanics maybe grouped under scalars and vectors.

Scalar Quantity :

A quantity is said to be scalar if it is completely defined by its magnitude alone.

Eg. Area, length, mass, Energy, power, volume Work etc.

Vector Quantity :-

A quantity is said to be vector if it is completely defined only when magnitude as well as direction are specified.

Eg. Force, moment, Momentum Displacement, Velocity acceleration etc.

FORCE SYSTEM

Force :-

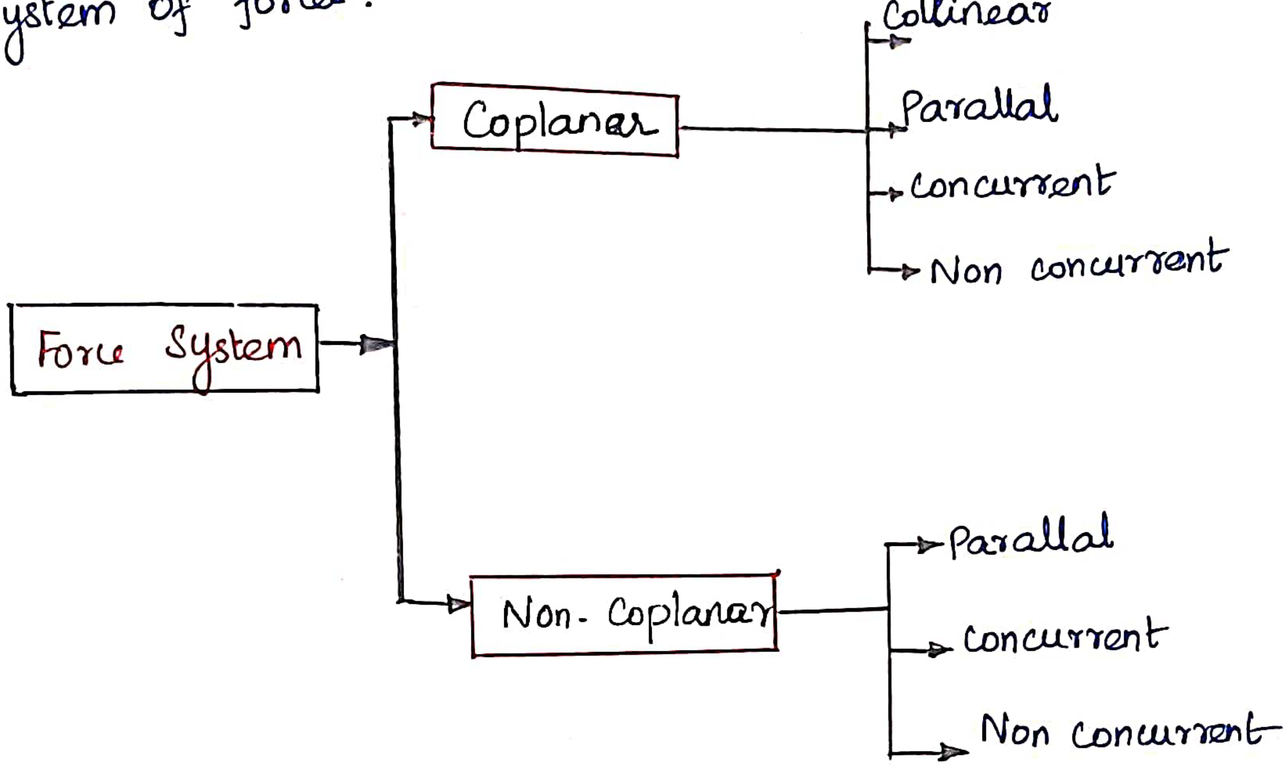
Force is defined as the cause of change in the state of motion of a particle or body. It is of course the product of mass of the particle and its acceleration.

$$\boxed{F = ma}$$

Force is the manifestation of action of one particle on the other. It is a vector quantity.

System of forces:-

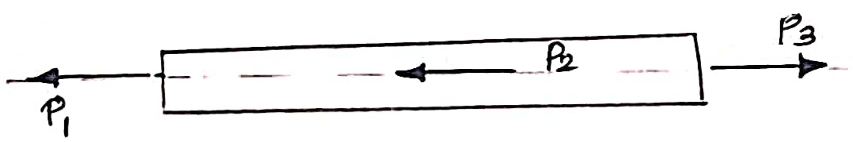
When a mechanics problem or system has more than one force acting, it is known as a 'force system' or 'system of forces'.



Force System.

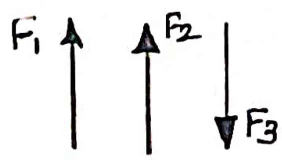
Collinear Force System:

When the lines of action of all the forces of as system act along the same line, this force system is called collinear force system.



Parallal Force System:

When the line of action of all the forces Parallal to one another is called parallal force system.



Characteristics of Force :-

A Force has following basic characteristics.

(i) Magnitude

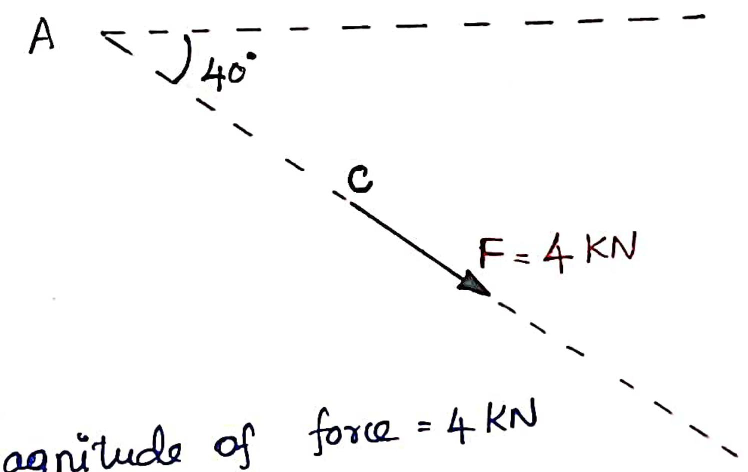
(ii) Direction

(iii) Point of application

(iv) Line of action.

Force is represented as a vector ie an arrow with its magnitude.

For Example



Here magnitude of force = 4 KN

direction is 40° with the horizontal in fourth quadrant

Point of application is 'C'

Line of action is 'AB'

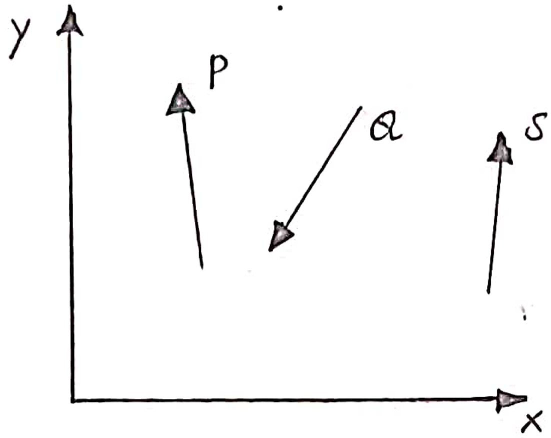
Smaller magnitude of forces are measured in newton (N) and larger magnitudes of forces are measured in Kilo Newton (kN)

Coplanar Force System:

When the line of action of a set of forces lie in a single plane is called coplanar force system.

Non Coplanar Force system:

when the line of action of all the forces do not lie in one plane, is called Non- Coplanar force system.



Concurrent force system :-

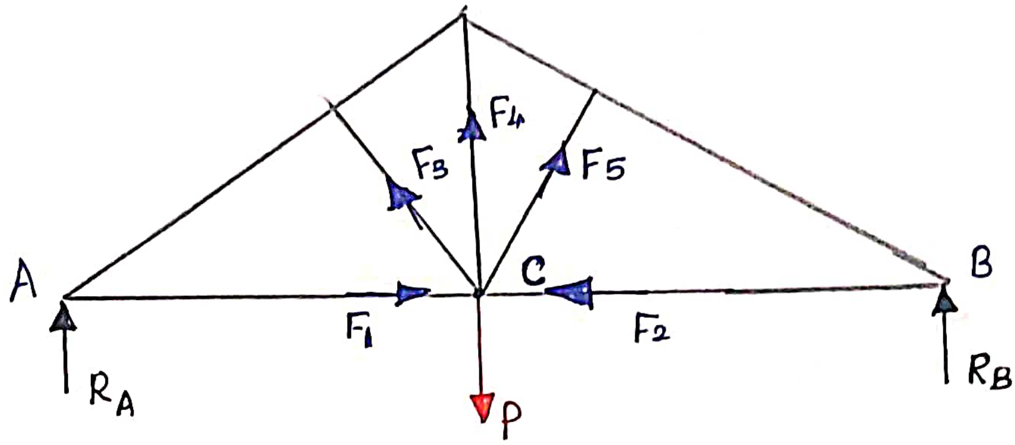
The forces when extended pass through a single point and the point is called point of concurrency. The lines of actions of all forces meet at the point of concurrency. Concurrent forces may or may not be coplanar.

Non concurrent force system :-

When the forces of a system do not meet at a common point of concurrency, this type of force system is called non concurrent force system. Parallal forces are examples of this type of force system. Non concurrent force system maybe coplanar or non coplanar.

Coplanar and Concurrent force System:

A force system in which all the forces lies in a single plane and meet at one point. For (eg) force acting at a joint of a roof truss.



P → External forces

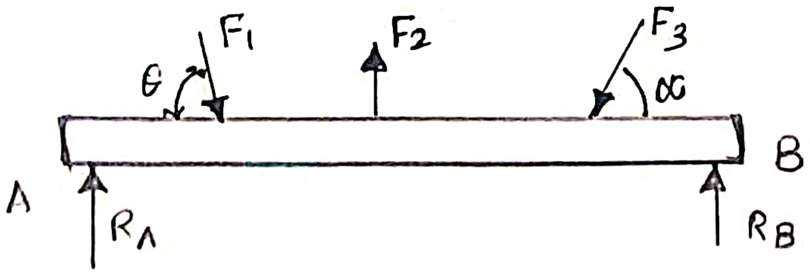
R_A & R_B → Reactions

F_1 to F_5 → Member forces (internal)

C = Point of concurrency.

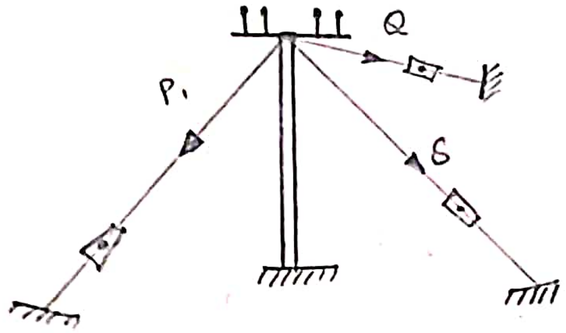
Coplanar non concurrent force system :-

These forces do not meet at a common points, however they lie in a single plane, for (eg) forces acting on a beam shown in figure.



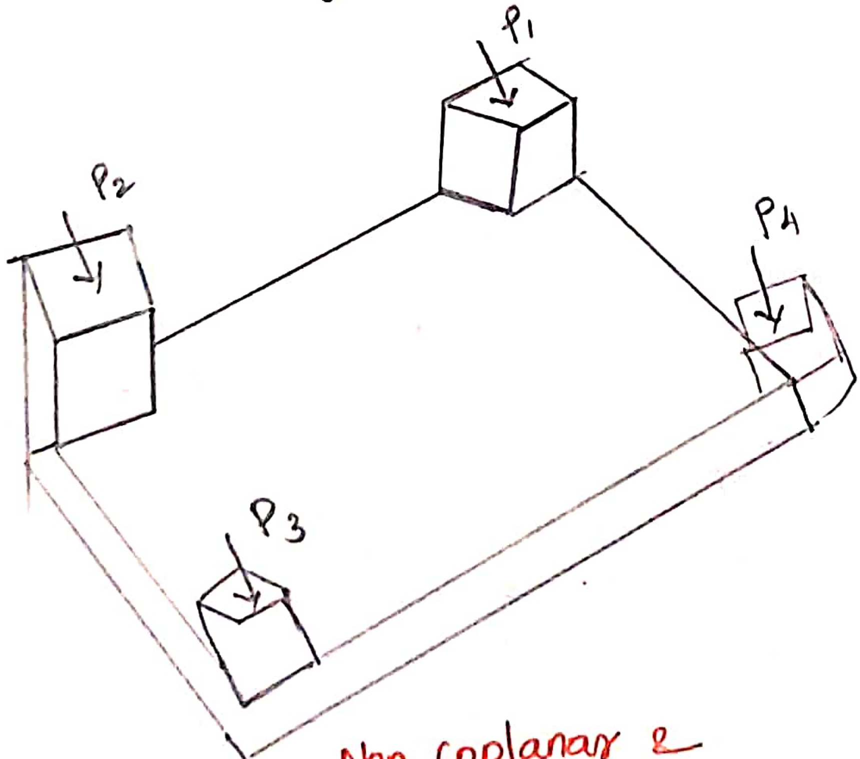
Non coplanar and Concurrent force System :-

In this system, the forces lie in a different planes but passes through a ~~signle~~ single point. Example is force acting at the top of an electric pole.



Non- coplanar and Non- concurrent force system :-

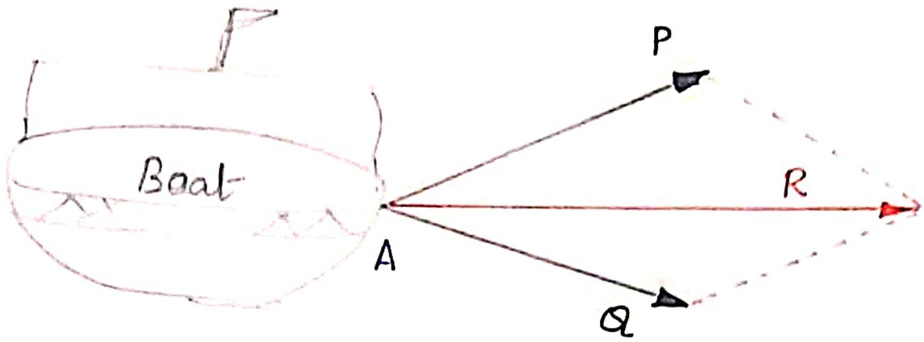
The forces which do not lie in a single plane and do not pass through a single point are known as non coplanar forces. Example is the loads transferred through Columns to the rectangular mat foundations.



Non coplanar & Non concurrent force system

Principle of Superposition of Forces :-

This principle states that, the combined effect of force system acting on a particle or a rigid body is the sum of effects of individual forces.

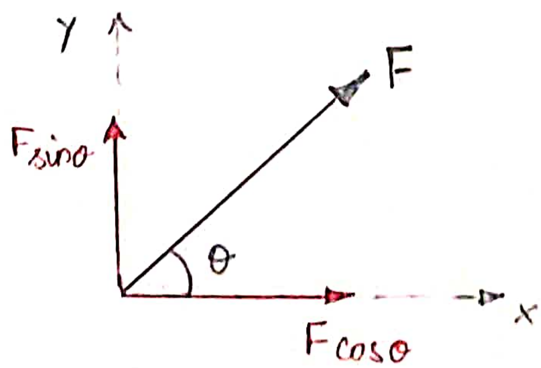


Resoultion of A Force into Compononents :-

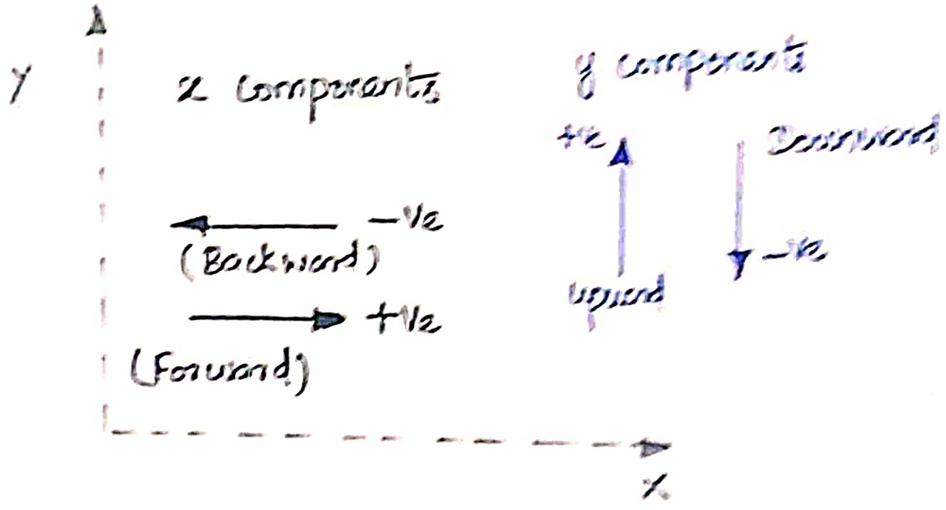
A Force F canbe resolved into (or replaced by) two componennts, which together produces the same effects that of force 'F'. These forces are called components of forces.

This process of replacing a force into its components is known as resolution of a force into components

If the two components are perpendicular to one another then they are known as rectangular components and when the components are inclined to each other, they are called inclined components.

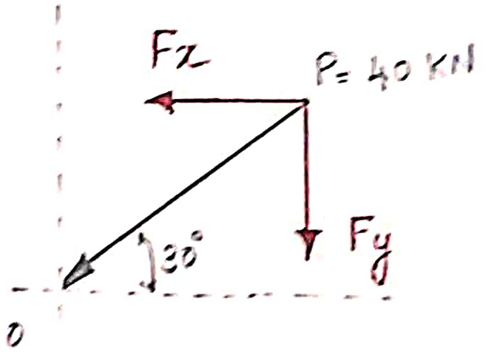


Sign Conversions :-



Problems on Resolution of Forces:-

1) Determine the components of force P = 40 KN along x and y axis, as shown in figure.



$F_x = F\cos\theta \quad = 40\cos 30^\circ$

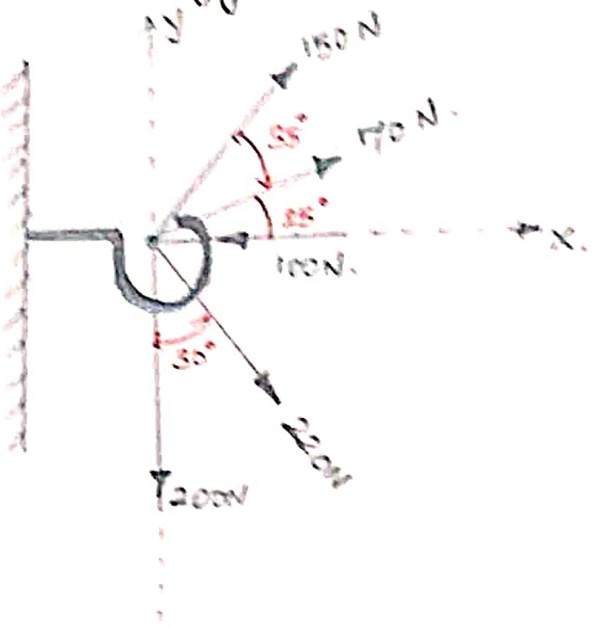
$F_y = F\sin\theta \quad = 40\sin 30^\circ$

$F_x = 34.64\ \text{KN}\ (\leftarrow)$

$F_y = 20.00\ \text{KN}\ (\downarrow)$

$$\boxed{\begin{aligned} F_x &= -34.64\ \text{KN} \\ F_y &= -20.00\ \text{KN} \end{aligned}}$$ Answer

2) Determine the x and y components of each forces shown in the figure.

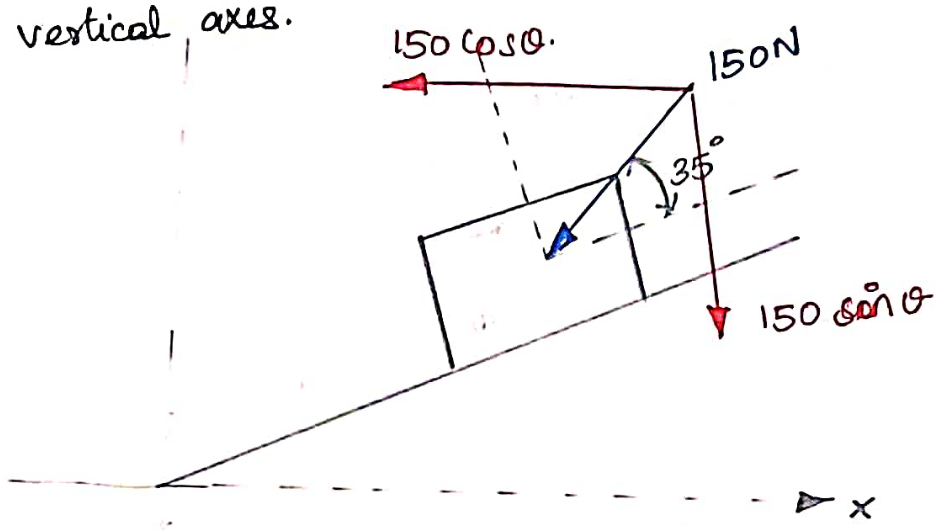


Solution :-

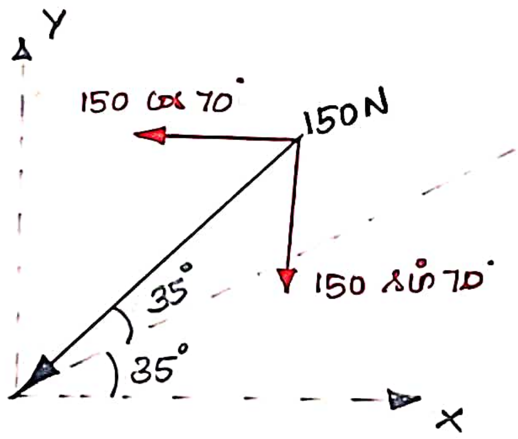
Force	Inclination with x axis	x component	y component
P	θ	$P\cos\theta$	$P\sin\theta$
150 N	$(25^\circ+35^\circ) = 60^\circ$	$150 \cos 60^\circ$ $= 75$ N ($\rightarrow$)	$150 \sin 60^\circ$ $= 129.90$ N ($\uparrow$)
70 N	25°	$70 \cos 25^\circ$ $= 63.44$ N ($\longrightarrow$)	$70 \sin 25^\circ$ 29.58 N ($\uparrow$)
100 N	0°	$100 \cos 0^\circ$ $= -100$ N ($\leftarrow$)	$100 \sin 0^\circ$ $= 0$
220 N	$(90^\circ - 30^\circ)$ $= 60^\circ$	$220 \cos 60^\circ$ $= 110$ N ($\rightarrow$)	$220 \sin 60^\circ$ $= -190.53$ N ($\downarrow$)
200 N	90°	$200 \cos 90^\circ$ $= 0$	$200 \sin 90^\circ$ $= -200$ N ($\downarrow$)

(17)

3) A force of 150N is acting on a block as shown in figure. Find the components of force along the horizontal and vertical axes.



Solution :-



$\theta = 70^\circ$

$P_x = 150 \cos 70^\circ = -51.30$ N (←)

$P_y = 150 \sin 70^\circ = -140.95$ N (↓)

↓

Resultant Forces

A resultant force is a single force, which produce, same affect so that of number of forces can produce is called "Resultant force".

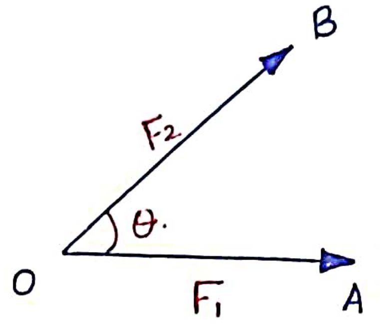
Composition of Forces:-

The process of finding out of the resultant force of given forces is called composition of force. A resultant force may be determined by following methods.

1. Parallelogram laws of forces (or) method
2. Triangular law of forces or Triangular method.
3. Polygonal law of forces or polygon method.

A) Parallelogram Method:-

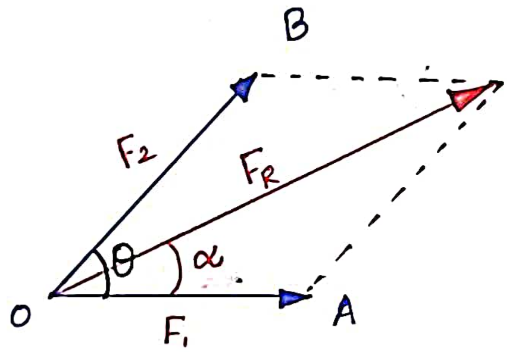
According to parallelogram method, 'If two forces (vectors) are acting simultaneously on a particle be represented (in magnitude and direction) by two adjacent sides of a parallelogram, their resultant force may represent (in magnitude & direction) by the diagonal of the parallelogram passing through the point.



OA = F_1

OB = F_2

θ = Angle between two forces.



F_R = Resultant force ⇒ Length of Diagonal of the parallelogram.

α = Direction of Resultant force

Analytically,

$$\boxed{F_R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}}$$

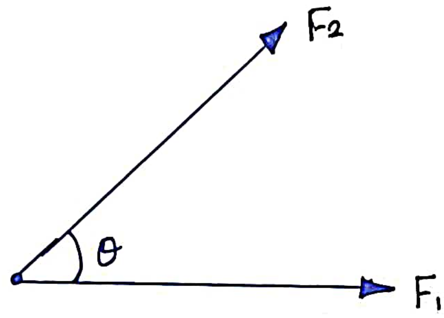
$$\boxed{\alpha = \tan^{-1}\left[\frac{F_2\sin\theta}{F_1 + F_2\cos\theta}\right]}$$

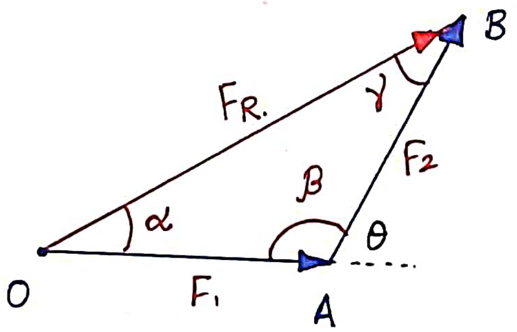
(OR)

$$\boxed{\alpha = \sin^{-1}\left[\frac{F_2\sin\theta}{F_R}\right]}$$

Triangle Method (or) Triangular Law of Forces :-

According to triangle law (or) trianglura method, If two forces acting simultaneously on a particle by represented in (magnitude and direction) by the two sides of a triangle taken in order their resultant is represented (in magnitude and direction) by the third side of triangle taken in opposite order.





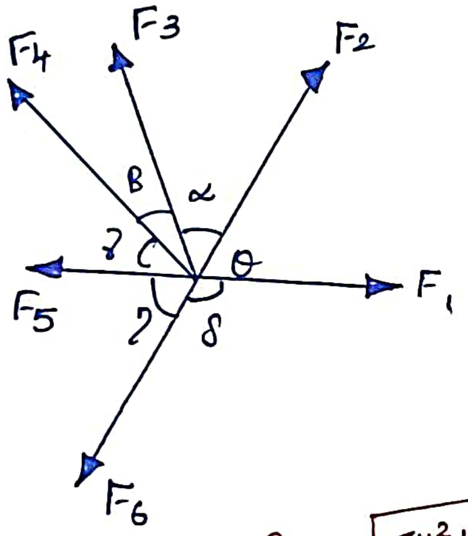
$$\boxed{F_R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\beta}}$$ Magnitude.

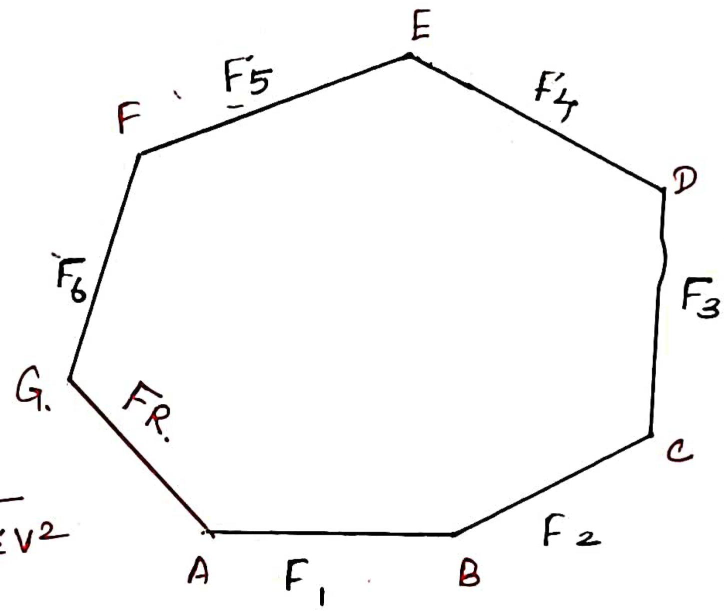
$$\beta = 180^\circ - \theta.$$

$$\boxed{\frac{F_1}{\sin\gamma} = \frac{F_2}{\sin\alpha} = \frac{F_R}{(\sin 180^\circ - \theta)}}$$ Direction of Resultant

Polygon Method:-

According to this method, "if more than two forces acting on a particle by represented by the side of a polygon taken in order, their resultant will be represented by closing side of the polygon in opposite direction.



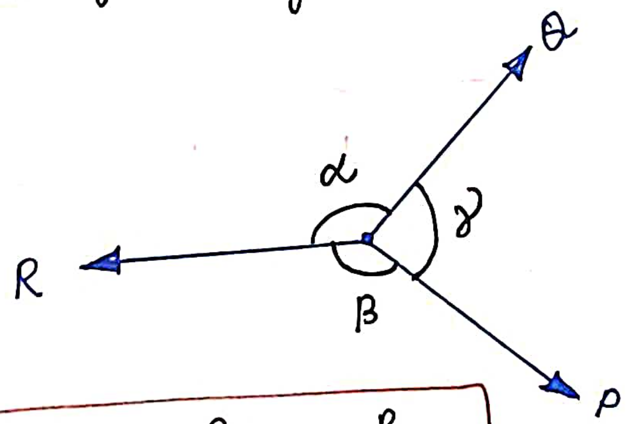


$$R = \sqrt{\Sigma H^2 + \Sigma V^2}$$

$$\tan\alpha = \frac{\Sigma V}{\Sigma H}$$

Lami's theorem:-

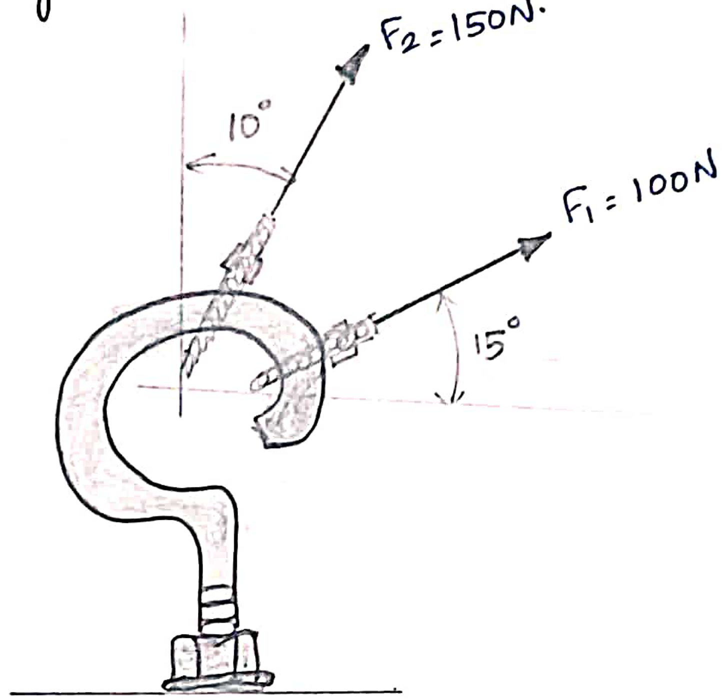
It states that, "If three forces acting at a point are in equilibrium, each force will be proportional to the sine of the angle between the other two forces.

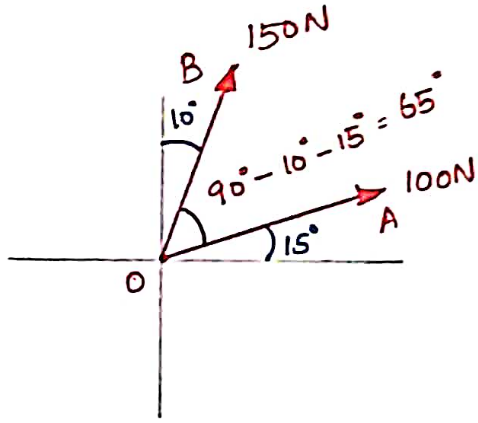


$$\boxed{\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}}$$

Problems on Resultant forces.

1) A screw eye is subjected to two forces F_1 and F_2 as shown in figure. Determine the magnitude and the direction of the resultant force by parallelogram by using analytical and grapical method.





Given :-

$F_1 = 100\ N$ $\quad F_2 = 150\ N.$

$\theta_1 = 15°$; $\theta_2 = 10°$

Required :-

Resultant force

Solution :-

θ = Angle between the forces

$= 90° - 10° - 15°$

$\boxed{\theta = 65°}$

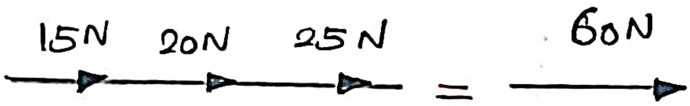
2) Find the resultant force of the collinear force System shown in figure.

Solution :-

Magnitude of resultant force $R = 15 + 20 + 25$

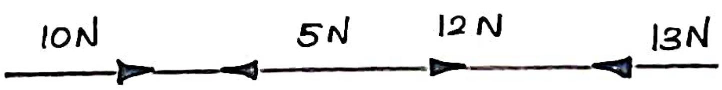
$\boxed{R = 60\,N \rightarrow}$

Answer.



Direction of resultant force and lines of action of resultant force are same. (Forward direction)

3) Find the resultant force of collinear force system shown in figure



Solution :-

Magnitude of resultant force

$R = 10 - 5 + 12 - 13$

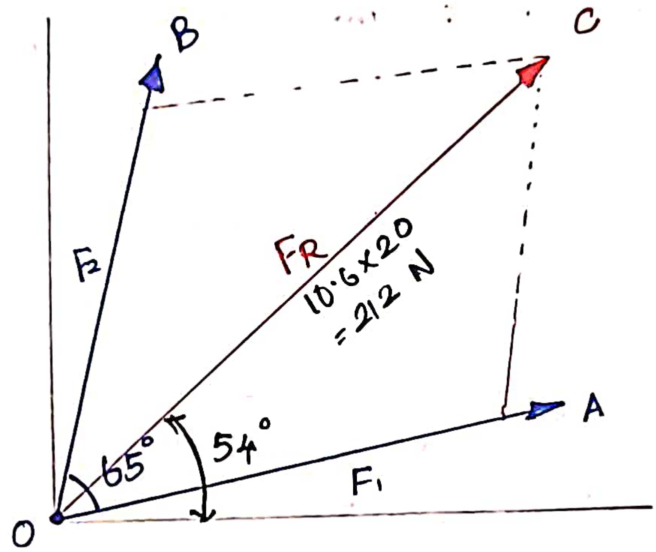
$\boxed{R = 4\,N \rightarrow}$ Answer.

Since R is positive, the direction of resultant force is towards right $\rightarrow$ (forward direction)

A) Graphically :-

Scale 20 N = 1 cm

Now draw parallelogram OABC with rule & protractor according to the scale,



By measuring

$OC = F_R = 10{\cdot}6 \text{ cm} \Rightarrow 10{\cdot}6 \times 20 = 212 \text{ N}.$

$\alpha = 54°$ with x axis.

B) Analytically :-

Resultant $R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

$\theta = 65°$

$F_1 = 100 \text{ N}$

$F_2 = 150 \text{ N}$

$R = \sqrt{100^2 + 150^2 + 2 \times 100 \times 150 \times \cos 65°}$

$\boxed{R = 212.55 \text{ N}}$ Ans.

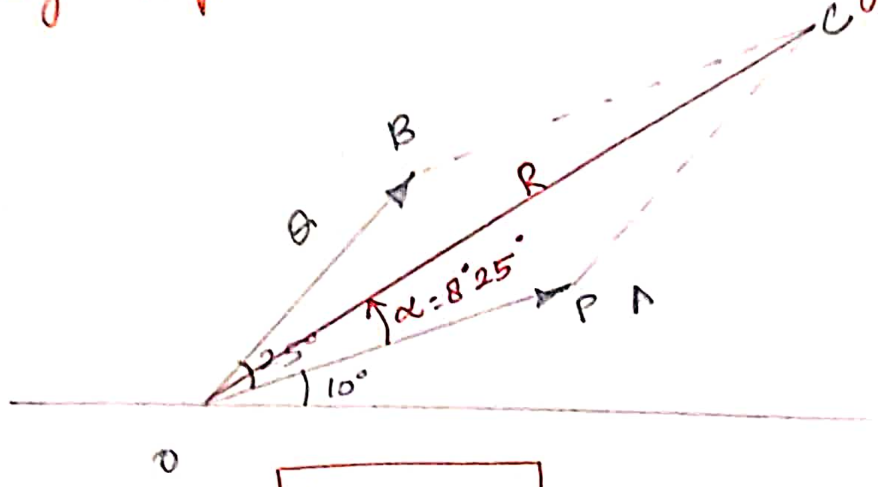
$\alpha = \sin^{-1}\left(\frac{F_2 \sin\theta}{R}\right)$

$\alpha = \sin^{-1}\left(\frac{150 \sin 65°}{212.55}\right) = 39.665°$ with F_1

$= 39.665 + 15° = \boxed{54.665° \text{ with x axis}}$

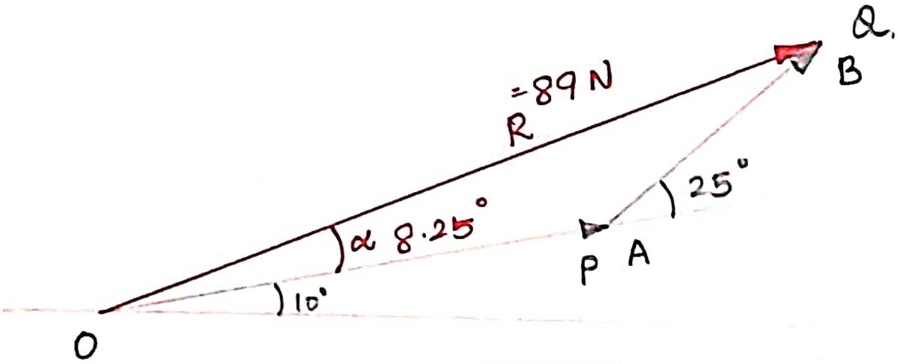
Ans

(ii) Using Graphical method of Parallelogram Law:-



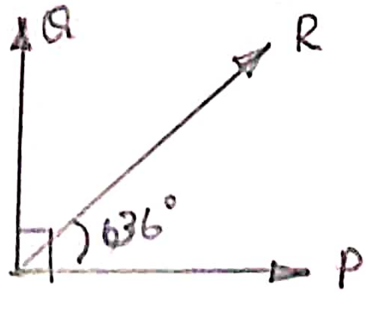
$$\boxed{\begin{array}{l} R = 89\ N \\ \alpha = 8.25^\circ \end{array}}$$ Answer

(iii) Using graphical method of Triangle Law:-



$$\boxed{\begin{array}{l} R = 89\ N \\ \alpha = 8.25^\circ \end{array}}$$ Answer.

5) The resultant of two concurrent forces is 1500 N and the angle between the forces is 90°. The resultant makes an angle of 36° with one of the force. Find the magnitude of each force.

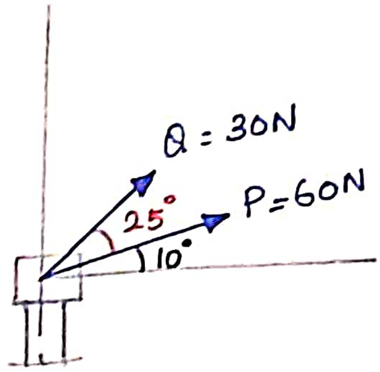


4) Two forces P and Q act on a bolt shown in figure. Determine their resultant,

(i) using Analytical Method

(ii) using graphical method of Parallelogram Law

(iii) using graphical method of Triangle law



Solution :-

$P = 60\,N$

$Q = 30\,N$

$\theta = 25°$

(i) using Analytical Method :-

Resultant $R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

θ = Angle between the forces P and Q

$$R = \sqrt{60^2 + 30^2 + 2 \times 60 \times 30 \times \cos 25°}$$

$\boxed{R = 88.1\,N}$ Ans.

Direction of resultant, $\tan\alpha = \dfrac{Q\sin\theta}{P + Q\cos\theta}$

$$= \frac{30\sin 25°}{60 + 30\cos 25°} = 0.145$$

$\alpha = \tan^{-1}(0.145)$

$\alpha = 8.27°$ with respect to the direction of P

Ans.

Given data :-

Resultant $R = 1500\,N$

Angle between forces $\theta = 90^\circ$

Angle made by resultant with one force $\alpha = 36^\circ$

Solution :-

using equation $\tan\alpha = \dfrac{Q\sin\theta}{P + Q\cos\theta}$

$$\tan 36^\circ = \frac{Q\sin 90^\circ}{P + Q\cos 90^\circ}$$

$$0.726 = \frac{Q \times 1}{P + Q \times 0} = \frac{Q}{P}$$

$$Q = 0.726\,P \quad \text{——— (1)}$$

using equation, $R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$$R^2 = P^2 + Q^2 + 2PQ\cos\theta$$

$$(1500^2) = P^2 + (0.726\,P)^2 + 2P \times 0.726P \times \cos 90^\circ$$

$$(1500)^2 = P^2 + 0.527\,P^2 + 0$$

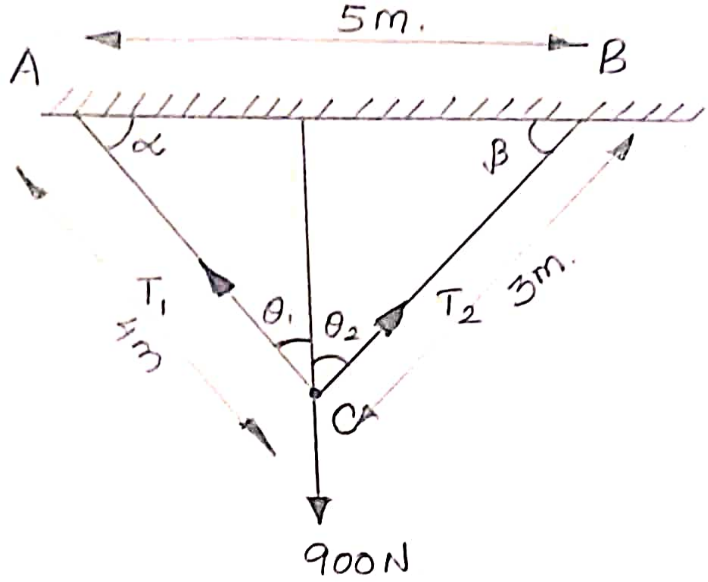
$$P = \sqrt{\frac{1500^2}{1.527}} = \frac{1500}{1.2357}$$

$\boxed{P = 1213.86\,N}$ Answer

$$Q = 0.726P = 0.726 \times 1213.86$$

$\boxed{Q = 881.26\,N}$ Answer.

6) A weight of 900N is supported by two chains of length 4 m and 3 m as shown in figure. Determine the tension in each cable.



Solution:-

Weight at C = 900 N

AC = 4 m

BC = 3 m

AB = 5 m.

In Δ^{le} ABC

$AC^2 + BC^2 = 4^2 + 3^2$

$= 16 + 9 = 25$

$AB^2 = AC^2 + BC^2 \Rightarrow \Delta^{le}$ ABC is right angled triangle

$\angle ACB = 90^\circ$

$\sin\alpha = \frac{BC}{AB} = \frac{3}{5} = 0.6$

$\alpha = 36^\circ 52'$

$\alpha + \beta = 90^\circ$

$\beta = 90^\circ - \alpha$

$= 90^\circ - 36.52 = 53^\circ 8'$

Let T_1 = Tension in chain AC

T_2 = Tension in chain BC

Δ^{le} ADC

$$\theta_1 = 90^\circ - \alpha = 90^\circ - 36.52 = 53.8'$$

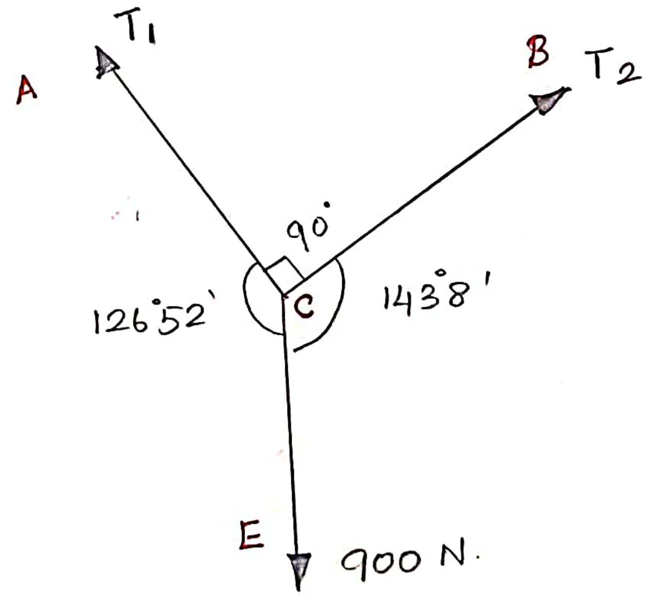
Δ^{le} BDC

$$\theta_2 = 90^\circ - \beta = 90^\circ - 53^\circ 8' = 36^\circ 52'$$

$$\angle ACE = 180^\circ - \theta_1 = 180^\circ - 53^\circ 8' = 126^\circ 52'$$

$$\angle BCE = 180^\circ - \theta_2 = 180^\circ - 36^\circ 52' = 143^\circ 8'$$

$$\angle ACB = 90^\circ$$



By using Lamis theorem

$$\frac{T_1}{\sin \angle BCE} = \frac{T_2}{\sin \angle ACE} = \frac{900}{\sin \angle ACB}$$

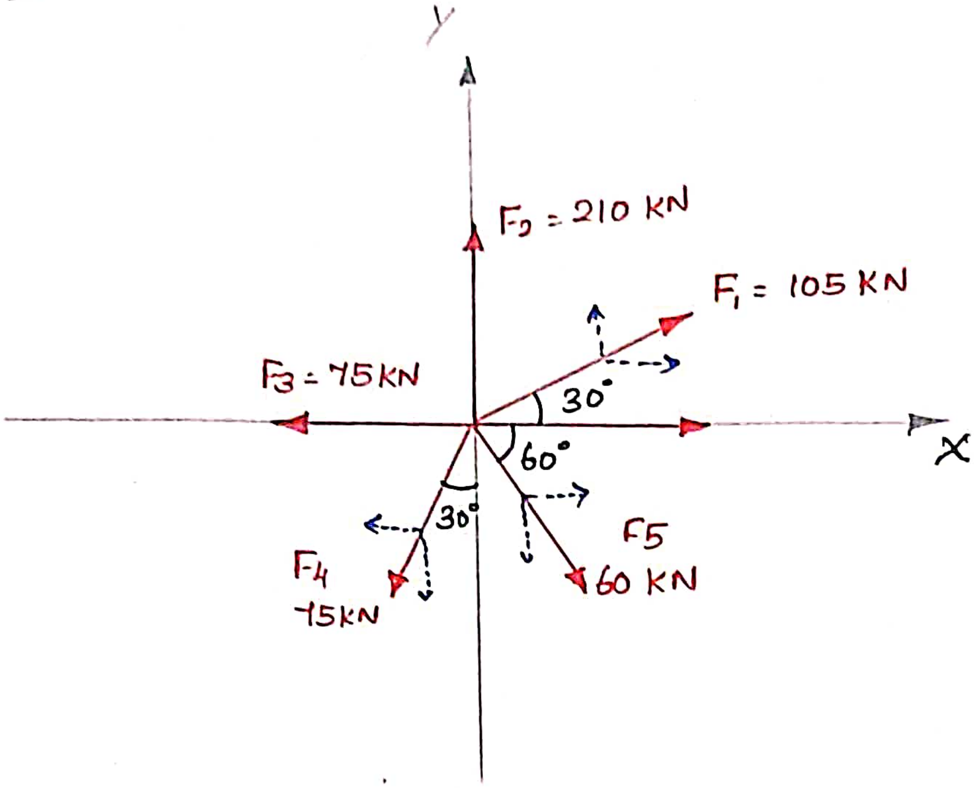
$$\frac{T_1}{\sin 143^\circ 8'} = \frac{T_2}{\sin 126^\circ 52'} = \frac{900}{\sin 90^\circ}$$

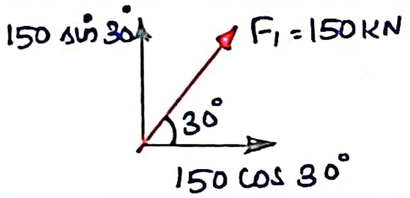
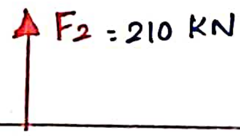
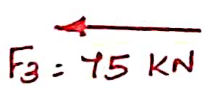
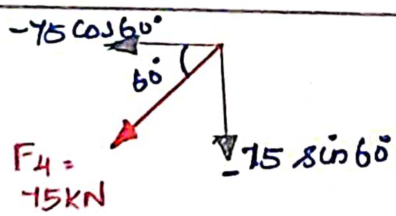
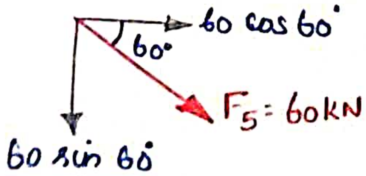
$$T_1 = 900 \times \sin 143^\circ 8' = 539.95 \text{ N}$$

$$T_2 = 900 \times \sin 126^\circ 52' = 720.00 \text{ N}$$

Answer.

7) If five forces act on a particle as shown in figure Determine the resultant force.



S.No	Force	H Horizontal Component	V Vertical component
1)		150 cos 30° = 90.93 KN	150 sin 30° = 52.5 KN
2.		210 cos 90° = 0	210 sin 90° = 210
3.		75 cos0° = −75	75 sin 0° = 0
4.		75 cos 60° = −37.5	75 sin 60° = −64.9
5.		60 cos 60° = 30	60 sin 60° = −51.96
		$\Sigma H = 8.43$	$\Sigma V = 145.59$

Resultant force is 520 N acting along the negative direction of y axis. (ie. downwards)

$\therefore \Sigma H = 0$

$\Sigma V = R = -520$ N.

$\Sigma H \Rightarrow 200 \cos 36.87 + P\cos\theta - 260\cos 67.38 - 360\cos 56.31$

$0 = 160 + P\cos\theta - 100 - 199.69$

$P\cos\theta = 139.69$ N ——— (1)

$\Sigma V \Rightarrow 200 \sin 36.87 - P\sin\theta - 260 \sin 67.38 + 360 \sin 56.31$

$-520 = 120 - P\sin\theta - 240 + 299.53$

$P\sin\theta = 699.53$ ——— (2)

(2) ÷ (1) $\Rightarrow \dfrac{P\sin\theta}{P\cos\theta} = \dfrac{699.53}{139.69} = 5.007$

$\tan\theta = 5.007$

$\theta = 78.7^\circ$

Substitute θ in eqn (1)

$P\cos 78.7^\circ = 139.69$

$P = \dfrac{139.69}{\cos 78.7}$

$P = 712.9$ N	Answer.
$\theta = 78.7^\circ$	

Resultant $\boxed{R = \sqrt{(\Sigma H)^2 + (\Sigma V^2)}}$

$$= \sqrt{(8.43)^2 + (145.59)^2}$$

$\boxed{R = 145.83 \text{ KN}}$ Answer

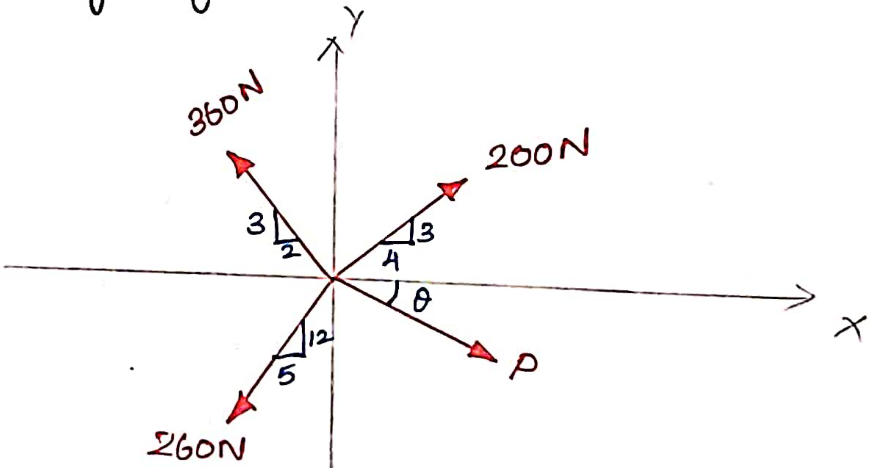
Direction of Resultant $\tan\alpha = \frac{\Sigma V}{\Sigma H}$

$$\tan\alpha = \frac{145.59}{8.43}$$

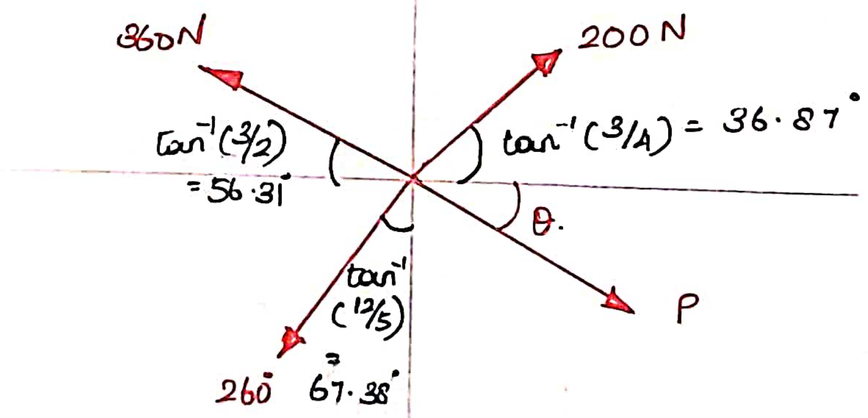
$\boxed{\alpha = 86.686^\circ}$ Answer.

---

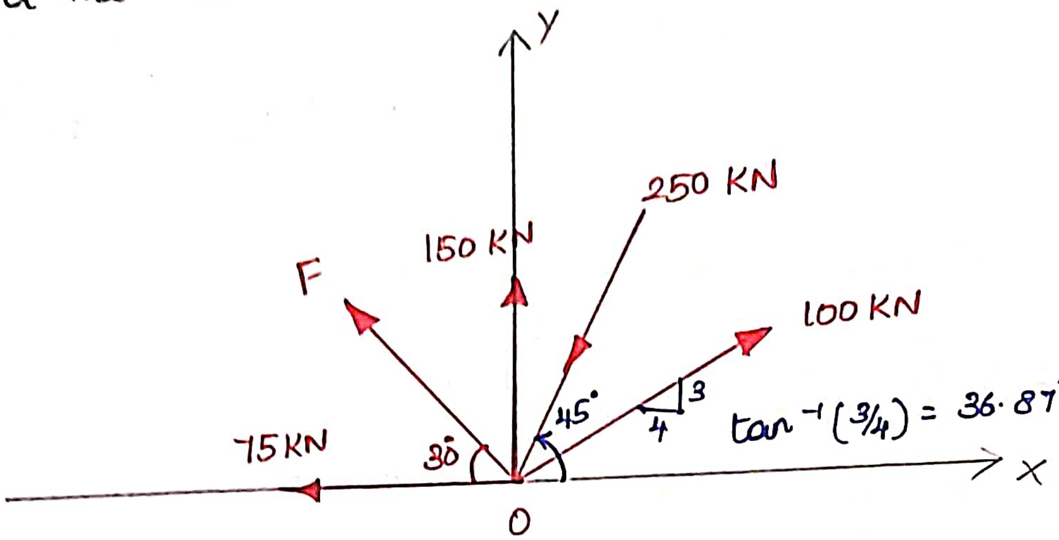
8) The resultant of the force system shown in figure is 520 N along negative direction of y axis. Determine P and θ.



Solution:-



9) If five forces act on a particle as shown in figure. and if the algebraic sum of horizontal component of all the forces is -300 KN. Calculate the magnitude of F and the resultant of all the forces.



Solution:-

Algebraic sum of horizontal component of all forces

$$\Sigma H = -300 \text{ KN}.$$

$$\Sigma H \Rightarrow -75 - F\cos 30° - 250\cos 45° + 100\cos 36.87°$$

$$-300 = -F\cos 30° - 171.8$$

$$F\cos 30° = 300 - 171.8$$

$$F = \frac{128.2}{\cos 30°} = +148.03 \text{ KN}.$$

$\boxed{F = 148.03 \text{ KN}}$ Answer.

Resolve the forces in Vertical direction

$$\Sigma V = 148.03\sin 30° + 150 - 250\sin 45° + 100\sin 36.87°$$

$$\Sigma V = 107.24 \text{ KN}.$$

$$\Sigma H = -300 \text{ KN}.$$

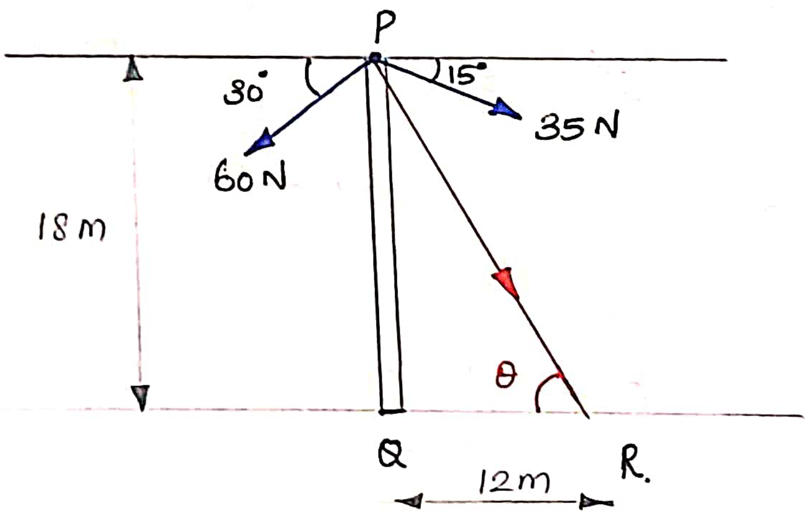
$$R = \sqrt{(-300)^2 + (107.24)^2}$$

$$\boxed{R = 318.59 \text{ KN}}$$ Answer

$$\tan\alpha = \frac{\Sigma V}{\Sigma H} = \frac{107.24}{-300} = -19.67°$$

$$\boxed{\alpha = -19.67° \text{ (or) } 340.33°}$$

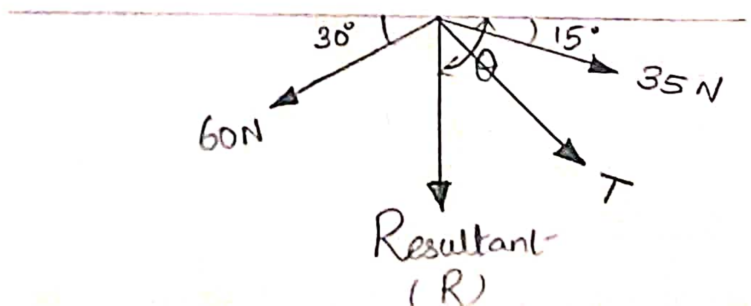
10) Two cables which have known tensions are attached to the top of the tower PQ. Third cable PR is used as a guy wire. Determine the tension in PR if the resultant of the forces exerted at 'P' by the three cables acts vertically, downwards.



Solution:

Resultant force is acting vertically downward

$\therefore$ $\Sigma H = 0$ & $\Sigma V = R$. (resultant)



In Δ^{le} PQR $\Rightarrow$ $\tan\theta = \frac{18}{12}$

$$\theta = \tan^{-1}\left(\frac{18}{12}\right) = 56^\circ 50^\circ$$

Resolve the given forces horizontally

$$\Sigma H = 35\cos 15^\circ - 60\cos 30^\circ + T\cos\theta = 0$$

$$33.8 - 51.96 + 0.555T = 0$$

$\boxed{T = 32.72\ N}$ Answer.

EQUILIBRIUM OF PARTCLE

When a body is said to be in equilibrium, then the resultant of all the forces acting on a particle is zero.

(OR)

If the algebraic sum of all the external forces is zero and also the algebraic moment of all the external forces about any point in their plane is zero.

Mathematically,

$\Sigma F = 0$ & $\rightarrow$ force law of Equilibrium

$\Sigma M = 0$ $\rightarrow$ Moment law of Equilibrium

Conditions of Equilibrium :-

$$\boxed{\Sigma F_x = 0\ ;\ \Sigma F_y = 0\ \&\ \Sigma M = 0}$$

Free body Diagram :-

A diagram or sketch of the body under consideration, is freed from the contact surface and all the forces acting on it (including reactions and actions) are drawn is called free body diagram.

Procedure of drawing Free body diagram :-

(i) Draw outline shape

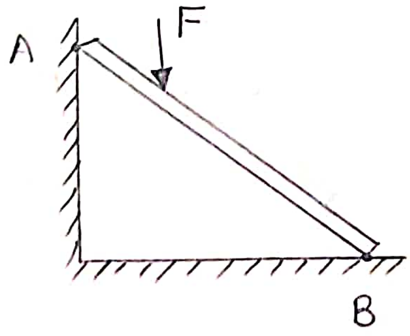
(ii) Show all forces

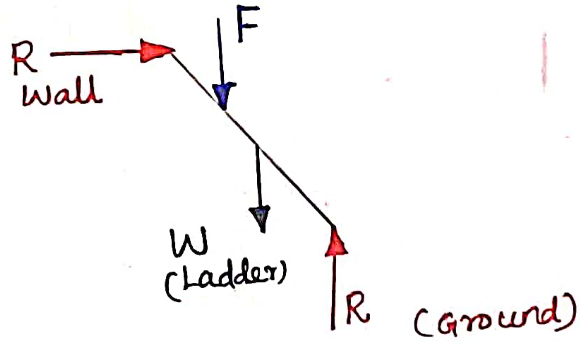
(iii) Identify each forces.

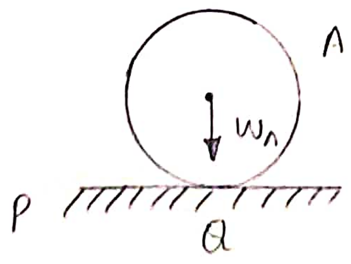
Method of Problem Solution :-

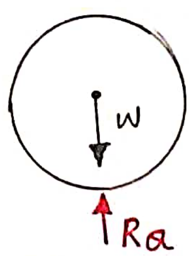
* Problem statement
* Freebody Diagrams
* Fundamental Principles
* Solution check
* Numerical Accuracy.

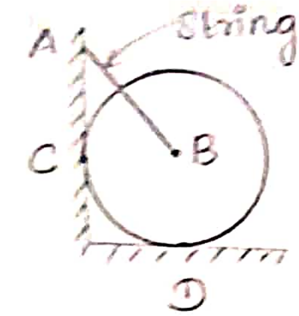
Free body Diagrams of Various force system :-

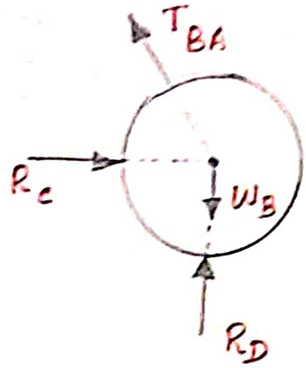


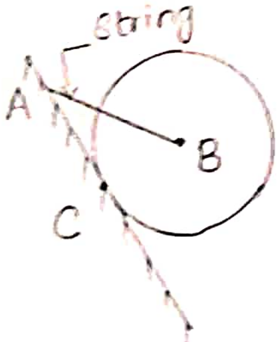


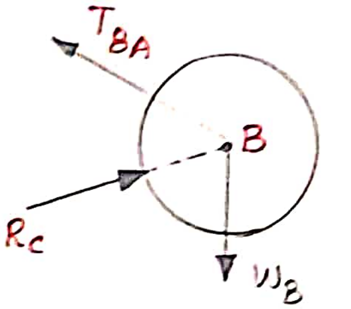


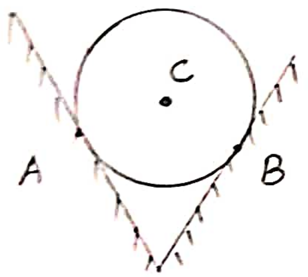


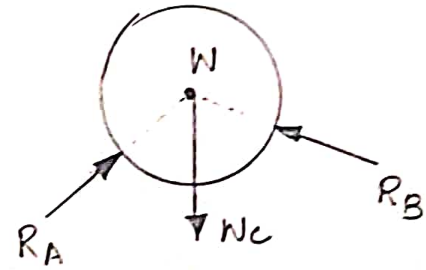

A
String
C
B
D

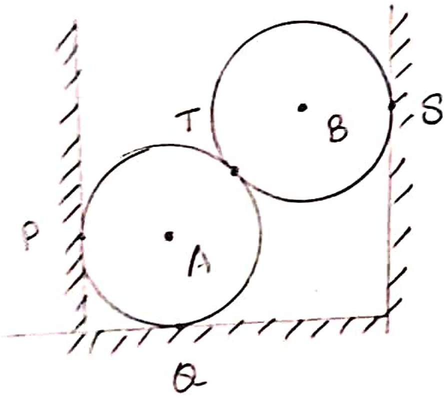

T_{BA}
R_c
W_B
R_D

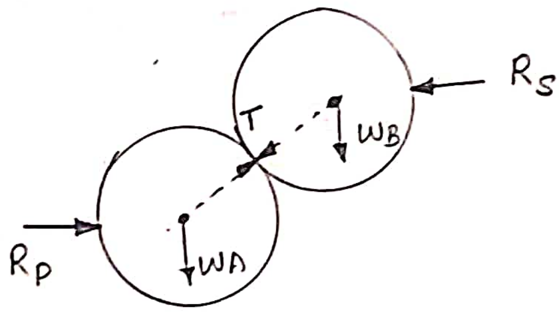

String
A
B
C

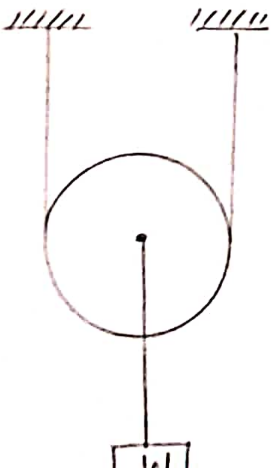

T_{BA}
B
R_c
W_B

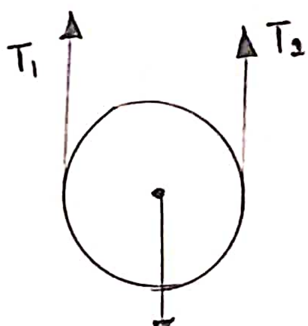

C
A
B


W
R_A
W_c
R_B


T
B
S
P
A
Q

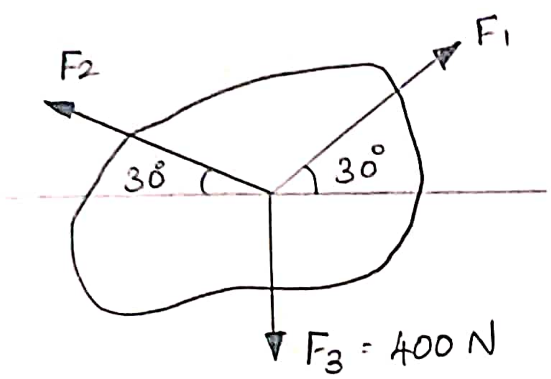

R_S
T
W_B
R_P
W_A

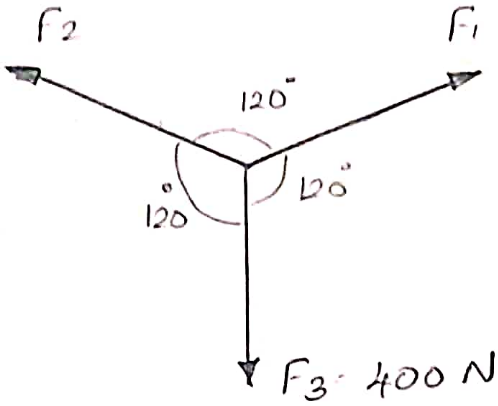

W


T_1
T_2
W.

PROBLEMS ON EQUILIBRIUM OF PARTICLE

1) Three forces F_1, F_2 and F_3 are acting on a body as shown in figure, and the body is in equilibrium. If the magnitude of force F_3 is 400 N, find the magnitude of force F_1 and F_2.





The Body is in Equilibrium

$$\Sigma F_x = 0 \quad \& \quad \Sigma F_y = 0$$

$$\Sigma F_x = 0 \Rightarrow \quad F_1 \cos 30^\circ - F_2 \cos 30^\circ = 0$$

$$F_1 - F_2 = 0$$

$$F_1 = F_2 \text{ ——— } (1)$$

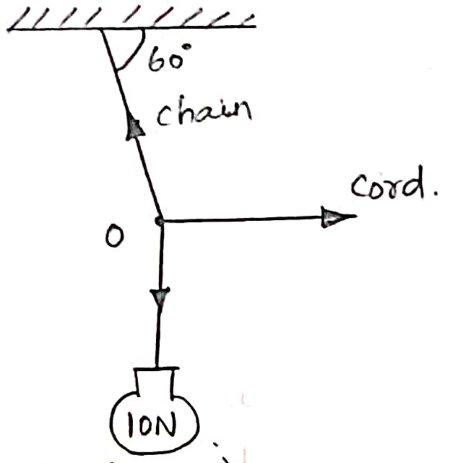
$$\Sigma F_y = 0 \Rightarrow \quad F_1 \sin 30^\circ + F_2 \sin 30^\circ - 400 = 0$$

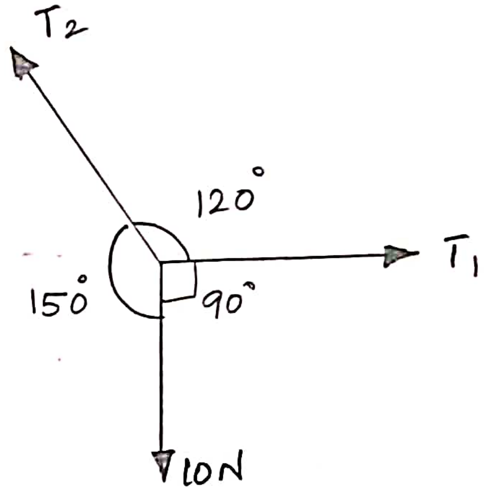
$$0.5 F_1 + 0.5 F_2 = 400 \qquad (F_1 = F_2)$$

$$\boxed{\begin{aligned} F_1 &= 400\ N \\ F_2 &= 400\ N \end{aligned}}$$

Answer!

2) A lamp weighing 10 N is suspended from the ceiling by a chain. It is pulled aside by a horizontal cord until the chain makes an angle of 60° with the cieling as shown in figure. Find the tensions in the cable by applying Lami's theorem.





Solution :-

By Applying Lamis theorem

$$\frac{T_1}{\sin 150^\circ} = \frac{T_2}{\sin 90^\circ} = \frac{10}{\sin 120^\circ}$$

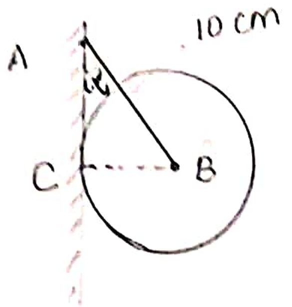
$$T_1 = 10 \times \frac{\sin 150^\circ}{\sin 120^\circ} = 5.774\ N.$$

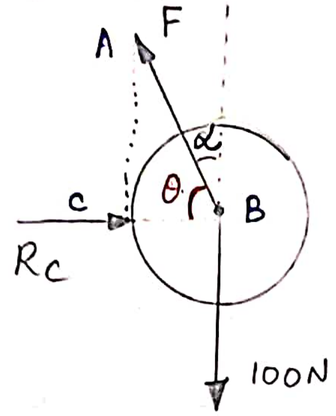
$$T_2 = 10 \times \frac{\sin 90^\circ}{\sin 120^\circ} = 11.547\ N.$$

$T_1 = 5.774$ N
$T_2 = 11.547$ N

Answers.

3) A circular roller of weight 100 N and radius 5 cm hangs by a tie rod AB = 10 cm and rests against a smooth vertical wall at C as shown in figure. Determine (i) the force F in the tie rod, (ii) the reaction R_c at point C.





$\Delta ABC \Rightarrow \quad AC = \sqrt{AB^2 + BC^2}$

$AC = \sqrt{10^2 + 5^2} = \sqrt{125}$

$AC = 15$ cm.

$\sin\alpha = \frac{BC}{AB} = \frac{5}{10} = 0.5 \Rightarrow \alpha\ 30^\circ$

$\theta = 90 - 30^\circ = 60^\circ$

Since the roller is in equilibrium,

$\Sigma F_x = 0 \Rightarrow \quad R_c - F\cos\theta = 0$

$R_c = F\cos\theta$ —— ①

$\Sigma F_y = 0 \Rightarrow \quad -100 + F\sin\theta = 0$ —— ②

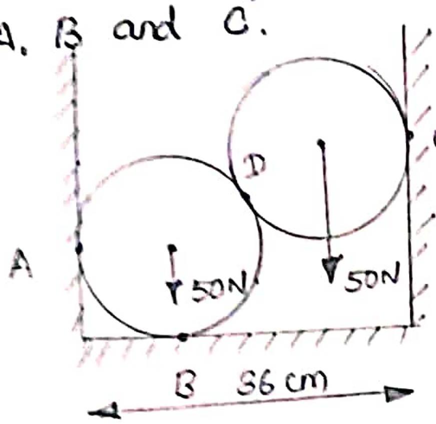
$F = \frac{100}{\sin\theta}$

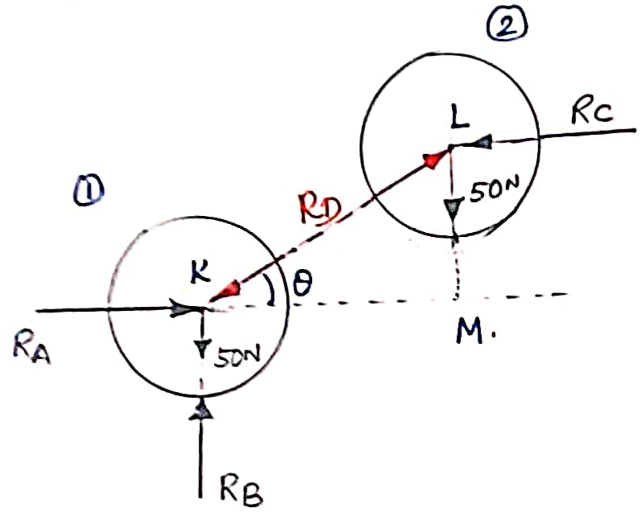
$F = \frac{100}{\sin 60^\circ} = \boxed{115.47\ N.}$ Ans.

$R_c = F\cos\theta = 115.47 \times \cos 60^\circ$

$\boxed{R_c = 57.73\ N}$ Answer

4) Two rollers each of weight 50 N and of radius 10 cm rest in a horizontal channel of width 36 cm as shown in figure. Find the reactions on the point of contact A, B and C. (41)





$$KL = 10\text{ cm} + 10\text{ cm} = 20\text{ cm}.$$

$$KM = 36 - (2 \times 10) = 16\text{ cm}.$$

$$\theta = \cos^{-1}\left(\frac{16}{20}\right) = 36.87^\circ$$

$$\boxed{\theta = 36.87^\circ}$$

Considering free body diagram of roller ②

$\Sigma F_x = 0 \Rightarrow \quad R_D \cos\theta - R_C = 0$

$R_D \cos 36.87 = R_C$ ——— ①

$\Sigma F_y = 0 \Rightarrow \quad R_D \sin\theta - 50 = 0$

$R_D \sin 36.87 = 50$

$\boxed{R_D = 83.33\text{ N}}$

$$\boxed{R_C = 66.66\ N}$$ Answer.

Considering free body diagram of roller ①

$\Sigma F_x = 0 \Rightarrow R_A - R_D \cos 36.87 = 0$

$R_A - 83.33 \cos 36.87 = 0$

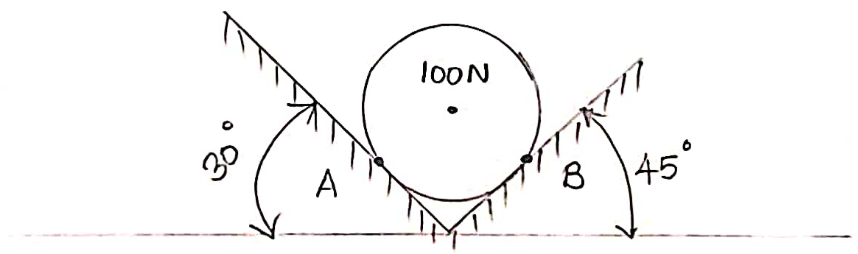
$$\boxed{R_A = 66.66\ N}$$ Answer

$\Sigma F_y = 0 \Rightarrow R_B - 50 - R_D \sin 36.87 = 0$

$R_B - 50 - 83.33 \sin 36.87 = 0$

$$\boxed{R_B = 99.99\ N}$$ Answer.

5) Determine the reaction at A and B.

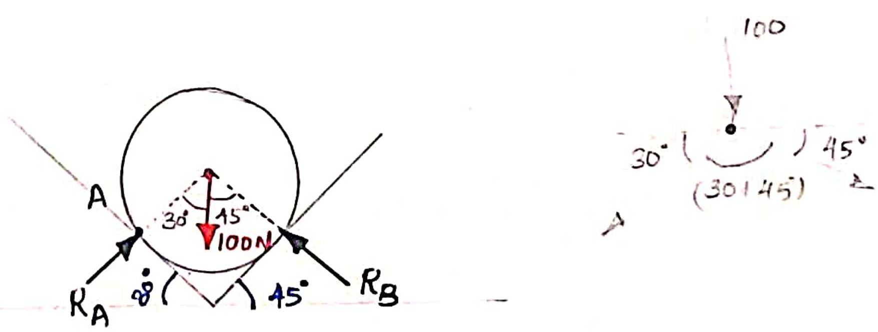


Given :-

Weight of block = 100 N

Angle made by the block are 30° & 45° on the plane.

Solution :-



By applying Lami's theorem

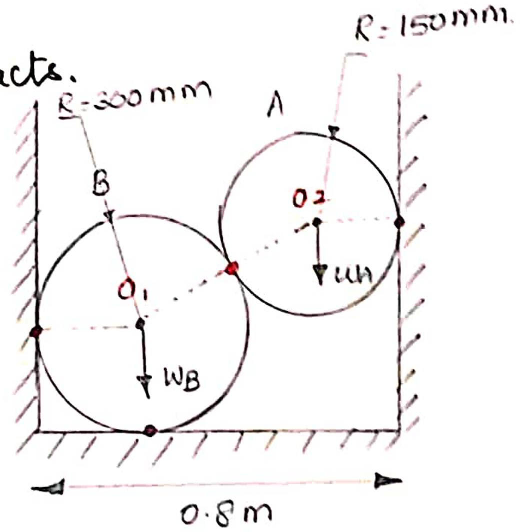
$$\frac{100}{\sin(90+45)} = \frac{R_A}{\sin 45} = \frac{R_B}{\sin 30}$$

$$\frac{R_A}{\sin 45} = \frac{100}{\sin(90+45)} \quad ; \quad \frac{R_B}{\sin 30^\circ} = \frac{100}{\sin 90+45}$$

$\boxed{R_A = 73.21\text{ N}}$ $\boxed{R_B = 51.76\text{ N}}$

Answers.

6) Two cylinderical pipes of radius 300 mm and 150 mm are lying in a trench with the longitudinal axis. The weight of the lower pipe is 1800 N and that of the upper pipe is 600 N. If the width of the trench is 0.8 m and the larger pipe is below the smaller one. Calculate the reaction at all points of contacts.



Given :-

$r_A = 150$ mm

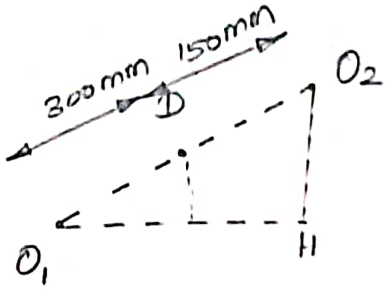
$r_B = 300$ mm

$W_A = 600$ N

$W_B = 1800$ N.

width b = 800 mm.

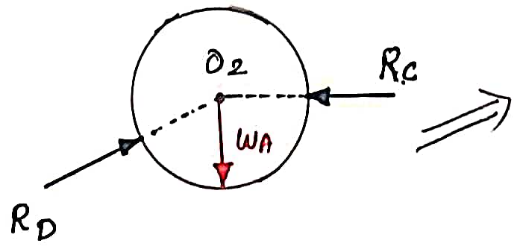
Free body diagram of 'A'



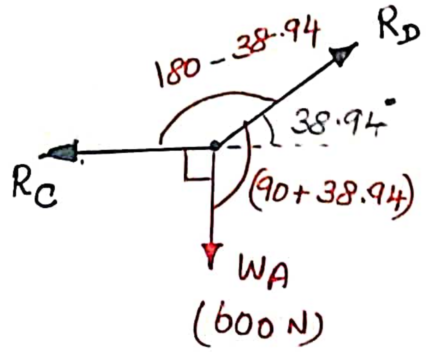
$O_1H = 800 - 300 - 150 = 350$ mm

$$\cos\theta = \frac{O_1H}{O_1O_2} = \frac{350}{450}$$

$$\theta = \cos^{-1}\left(\frac{350}{450}\right) = 38.94^\circ$$



$\Rightarrow$



Applying Lami's theorem at O_2

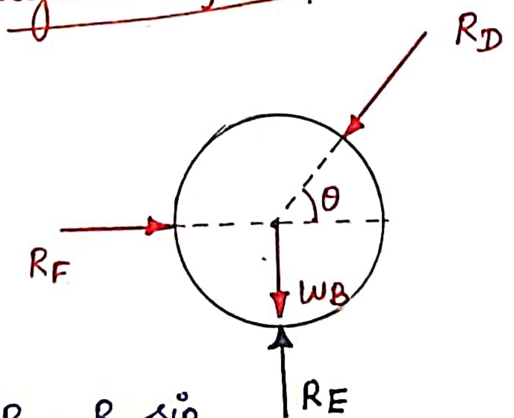
$$\frac{R_C}{\sin(90+38.94)} = \frac{600}{\sin(180-38.94)} = \frac{R_D}{\sin 90^\circ}$$

$$R_D = \frac{600}{\sin(180-38.94)} \Rightarrow \boxed{R_D = 954.6\ N}$$

$$R_C = \frac{600}{\sin(180-38.94)} \times \sin(90+38.94) \Rightarrow \boxed{R_C = 742.52 N}$$

Answers.

Free body diagram of 'B'



$\Sigma F_y = 0$

$R_E - R_D \sin 38.94^\circ - W_B = 0$

$R_E = 954.6 \sin 38.94^\circ + 1800 = 2400$ N.

$\boxed{R_E = 2400\ N}$ Answer

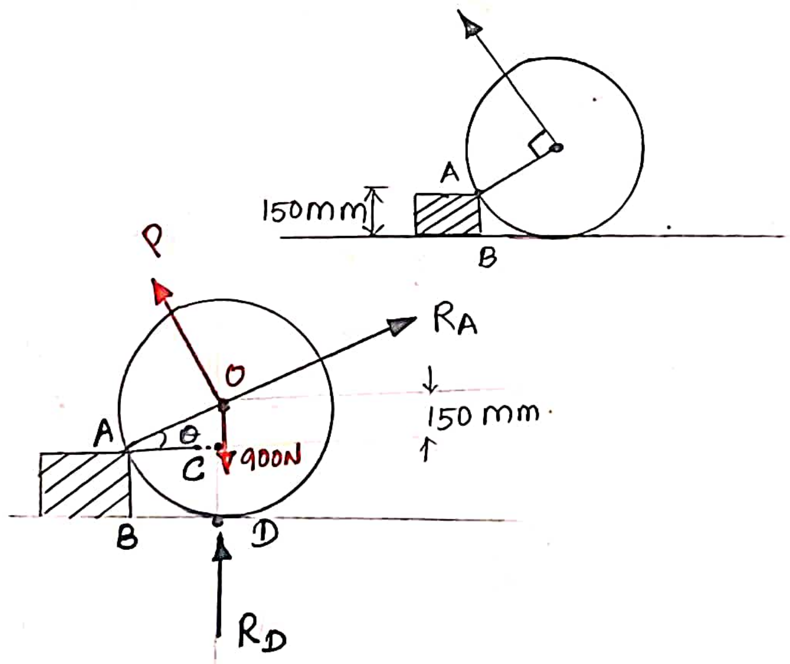
$\Sigma F_x = 0$

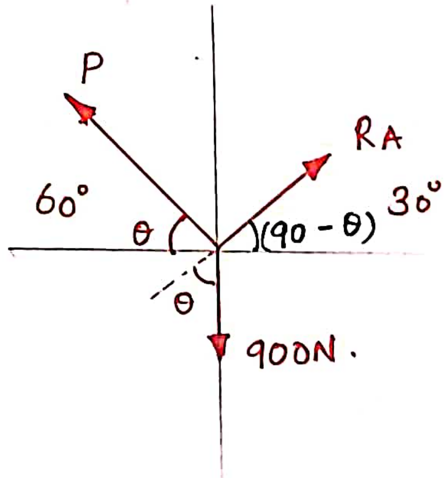
$$R_F - R_D \cos 38.94^\circ = 0$$

$$R_F = 954.6 \cos 38.94 = 742.46\ N$$

$$\boxed{R_F = 742.46\ N.}$$ Answer.

7) A uniform wheel 600 mm in diameter rests against a rigid rectangular block of 150 mm thick as shown in figure. Find the least pull P, through the centre of the wheel in order to just turn the wheel over the corner of the block. All surfaces are smooth. Find also the reactions of the block. The wheel weighs 900 N.





In right angle Δ^{le} AOC

$$OC = \text{Radius} - AB$$

$$= 300 - 150\ mm$$

$$OC = 150\ mm$$

$$AO = \text{radius} = \frac{600}{2} = 300\ mm$$

$$\cos\theta = \frac{OC}{OA} = \frac{150}{300} \quad \theta \Rightarrow \cos^{-1}\left(\frac{150}{300}\right) = 60^\circ$$

$\therefore$ The angle R_A with horizontal is $(90^\circ - 60^\circ) = 30^\circ$

IIIly angle of P with horizontal is $= \theta = 60^\circ$

Using Lami's theorem :-

$$\frac{P}{\sin 120^\circ} = \frac{R_A}{\sin 150^\circ} = \frac{900}{\sin 90^\circ}$$

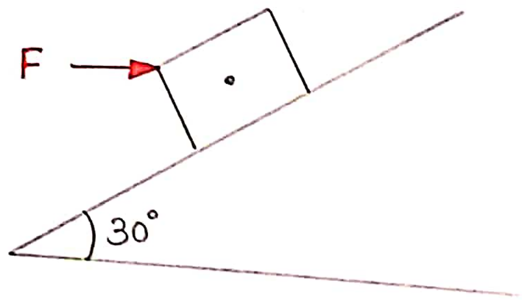
$$P = \frac{900}{\sin 90^\circ} \times \sin 120^\circ \boxed{= 779.4\ N.}$$

Answers.

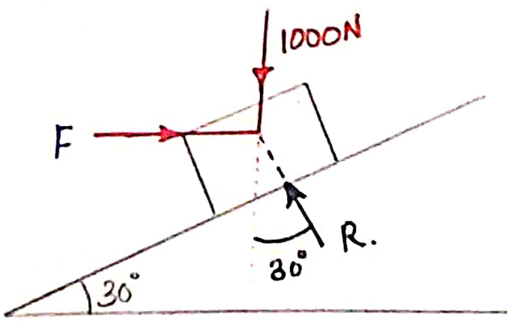
$$R_A = \frac{900}{\sin 90^\circ} \times \sin 150^\circ = \boxed{450\ N.}$$

---

8) A box weighing 1000 N is held at rest on a smooth inclined plane, at 30° to the horizontal by the application of a horizontal force F as shown in figure. Determine the value of 'F'



Free body diagram :-



$\Sigma F_x = 0$

$F - R\cos 60^\circ = 0$

$F - 0.5R = 0$ —— ①

$\Sigma F_y = 0$

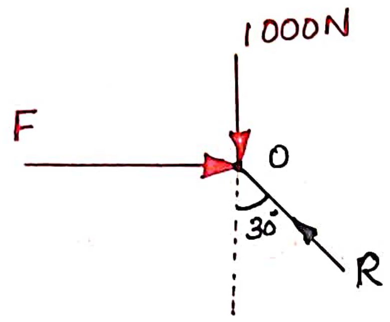
$-1000 + F\overset{0}{\cancel{\sin 0}} + R\sin 60^\circ = 0$

$0.866 R = 1000$ —— ②

$$0.866\,R = 1000$$

$$\boxed{\begin{array}{l} R = 1154.7\ N \\ F = 577.35\ N. \end{array}}$$ Answers.

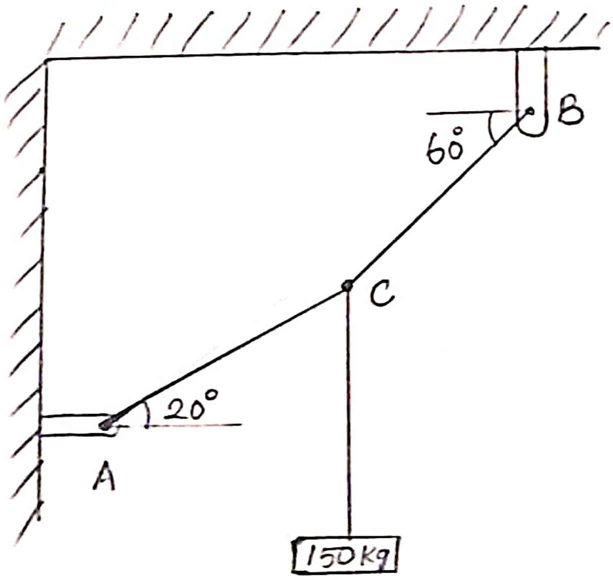
Alternatively (using Lami's theorem)



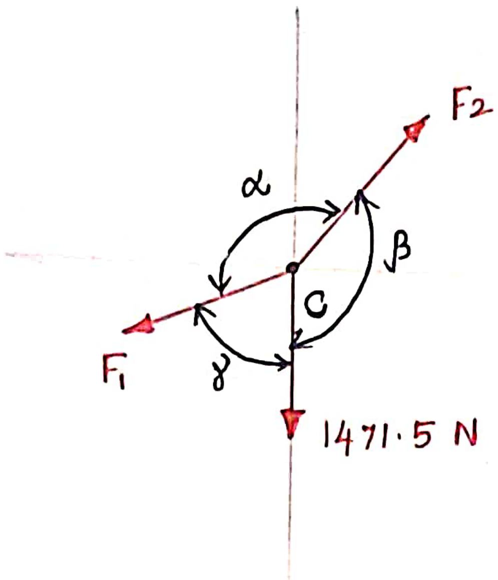
$$\frac{1000}{\sin(90^\circ + 30^\circ)} = \frac{F}{\sin(180^\circ - 30^\circ)} = \frac{R}{\sin 90^\circ}$$

$$\boxed{\begin{array}{l} F = 577.35\ N \\ R = 1154.7\ N. \end{array}}$$ Answers.

(9) Two cables are tied together at 'C' and loaded as shown in figure. Determine the tension in cables AC and BC



Free body diagram :-



$150\ Kg = 150 \times 9.81\ N$
$= 1471.5\ N.$

$\alpha = 20^\circ + 90^\circ + 30^\circ = 140^\circ$

$\beta = 60^\circ + 90^\circ = 150^\circ$

$\gamma = 70^\circ$

using Lami theorem

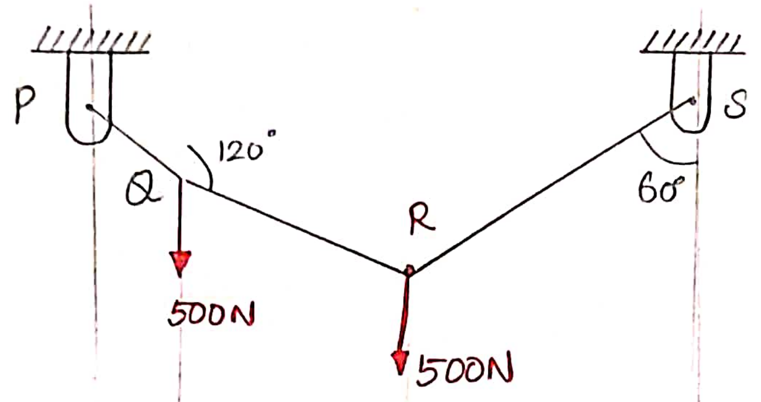
$$\frac{F_1}{\sin\beta} = \frac{F_2}{\sin\gamma} = \frac{1471.5}{\sin\alpha}.$$

$$\frac{F_1}{\sin 150^\circ} = \frac{F_2}{\sin 70^\circ} = \frac{1471.5}{\sin 140^\circ}$$

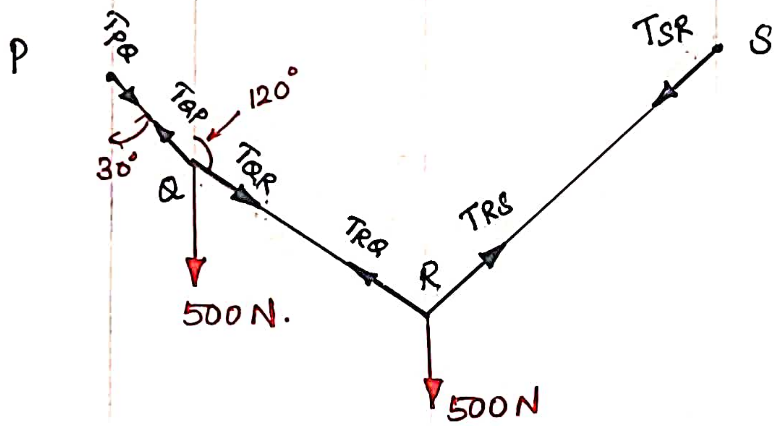
Solving $\boxed{F_1 = 1144.62\ N \quad F_2 = 2151.19\ N.}$ Answers.

---

10) A strings PQRS attached to two fixed points P and S has two equal weights of 500 N attached to it Q and R. The weights rest with the portions PQ and RS inclined at angles of 30° and 60° respectively, to the vertical. Find the tensions in the portions PQ, QR and RS of the string, if the inclination of the portion QR with the vertical is 120°



Freebody Diagram :-



Since the system is in equilibrium.

$T_{PQ} = T_{QP}$; $T_{QR} = T_{RQ}$ $T_{RS} = T_{SR}$.

Applying Lamis theorem for joint Q.

Angle b/w T_{QP} & T_{QR} $= 30^\circ + 120^\circ = 150^\circ$

T_{QR} & weight $= 180^\circ - 120^\circ = 60^\circ$

T_{QP} & Weight $= 180^\circ - 30^\circ = 150^\circ$

$$\frac{500}{\sin 150^\circ} = \frac{T_{QP}}{\sin 60^\circ} = \frac{T_{QR}}{\sin 150^\circ} \Rightarrow \text{Solving} \quad \boxed{\begin{aligned} T_{QP} &= 866N \\ T_{QR} &= 500N \end{aligned}}$$

Applying Lamis theorem for joint R

Angle b/w T_{RQ} & T_{RS} $= 60^\circ + 60^\circ = 120^\circ$

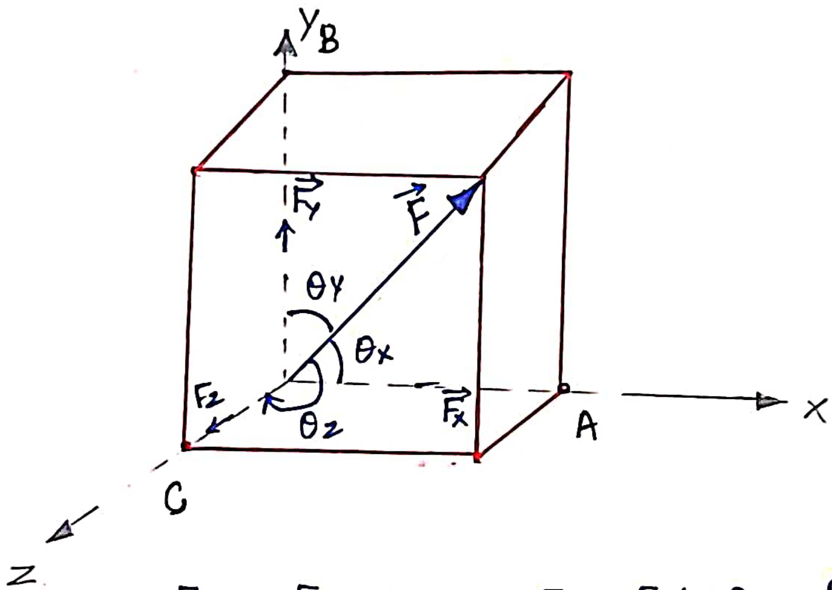
T_{RQ} & weight $= 180 - 60 = 120^\circ$

$$\frac{500}{\sin 120} = \frac{T_{RQ}}{\sin 120} = \frac{T_{RS}}{\sin 120}$$

Solving $\boxed{\begin{array}{l} T_{RQ} = 500\,N \\ T_{RS} = 500\,N \end{array}}$ Answer.

FORCE IN SPACE

(3 Dimensions)



$F_x = F\cos\theta_x$; $F_y = F\cos\theta_y$; $F_z = F\cos\theta_z$

Let $\vec{i}$, $\vec{j}$ & $\vec{k}$ are the unit vectors in respective direction

$$\boxed{\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}}$$

$$\vec{F} = F.\vec{n}_F$$

Magnitude of the force $\vec{F}$ is

$$|\vec{F}| = \boxed{F = \sqrt{F_x^2 + F_y^2 + F_z^2}}$$

Direction Cosines :-

θ_x, θ_y & θ_z.

Let $\cos\theta_x = l$; $\cos\theta_y = m$ & $\cos\theta_z = n$

then, $\vec{F} = F(l\vec{i} + m\vec{j} + n\vec{k})$

$\therefore$ $\boxed{l^2 + m^2 + n^2 = 1}$

also $\boxed{\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1}$

Unit Vector :-

$$\vec{n}_F = \frac{F_x\vec{i} + F_y j + F_z K}{\sqrt{F_x^2 + F_y^2 + F_z^2}}$$

Position Vector :-

$$\vec{r} = xi + yj + zk$$

$$|\vec{r}| = r = \sqrt{x^2 + y^2 + z^2}$$

Force defined by its Magnitude and Two points on its line of actions :-

$A \Rightarrow (x_1, y_1, z_1)$

$B \Rightarrow (x_2, y_2, z_2)$

$$\vec{AB} = (x_2 - x_1)\vec{i} + (y_2 - y_1)\vec{j} + (z_2 - z_1)\vec{k}$$

Distance b/w A & B

$$AB = |\vec{AB}| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

$$\vec{F} = F.\ \vec{n}F.$$

$$\vec{F} = \frac{F\left[(x_2-x_1)\vec{i} + (y_2-y_1)\vec{j} + (z_2-z_1)\vec{k}\right]}{\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}}$$

$$F_x = \frac{F(x_2-x_1)}{\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}}$$

$$F_y = \frac{F(y_2-y_1)}{\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}}$$

$$F_z = \frac{F(z_2-z_1)}{\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}}$$

Angle of inclination :-

$$\cos\theta_x = \frac{(x_2-x_1)}{\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}}$$

$$\cos\theta_y = \frac{(y_2-y_1)}{\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}}$$

$$\cos\theta_z = \frac{(z_2-z_1)}{\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}}$$

Resultant Force in space :-

$$\vec{R} = R_x \vec{i} + R_y \vec{j} + R_z \vec{k}$$

$R_x = \Sigma F_x$

$R_y = \Sigma F_y$

$R_z = \Sigma F_z$

$$\boxed{R = \sqrt{R_x^2 + R_y^2 + R_z^2}}$$

Direction of resultant force :-

$$R_x = R \cos\theta_x \Rightarrow \theta_x = \cos^{-1}\left(\frac{R_x}{R}\right)$$

|||ly $\theta_y = \cos^{-1}\left(\frac{R_y}{R}\right)$

$$\theta_z = \cos^{-1}\left(\frac{R_z}{R}\right)$$

Equilibrium of Particle in space :-

A particle is subjected to force system in space is said to be in equilibrium when the resultant force is zero.

$$\boxed{\vec{R} = R_x \vec{i} + R_y \vec{j} + R_z \vec{k} = 0}$$

ie $R_x = 0$ $R_y = 0$ & $R_z = 0$

Equation of Equilibrium for a Particle when subjected to forces in space :-

$$\Sigma F_x = 0$$

$$\Sigma F_y = 0$$

$$\Sigma F_z = 0$$

Transmissibility of Forces :-

It states that, "The state of rest or motion of a rigid body is unaltered if a force acting on a body is replaced by another force of same magnitude and direction, but acting anywhere on the body along the line of action of the replaced force."

Problems on Particle in space / Equilibrium.

1) A force 125 N makes an angle of 30°, 60° and 120° with x, y & z axis. Find the force Vector.

Given data :-

$F = 125\ N$

$\theta_x = 30°$; $\theta_y = 60°$ & $\theta_z = 120°$

Solution :-

$$\vec{F} = F_x \vec{i} + F_y \vec{j} + F_z \vec{k}$$

$$F_x = F \cos\theta_x = 125 \cos 30° = 108.25\ N.$$

$$F_y = F \cos\theta_y = 125 \cos 60° = 62.5\ N$$

$$F_z = F \cos\theta_z = 125 \cos 120° = -62.5\ N.$$

$$\boxed{\vec{F} = 108.25\, \vec{i} + 62.5\, \vec{j} - 62.5\, \vec{k}}$$

Answer

2) The components of force $\vec{F}$ = $F_x = 225$ N, $F_y = -300$N $F_z = 450$ N. Determine its magnitude 'F' and angle made by F with three co-ordinate axes.

Given :-

$F_x = 225$ N ; $F_y = -300$ N ; $F_z = 450$ N.

Solution :-

Magnitude of force $F = \sqrt{F_x^2 + F_y^2 + F_z^2}$.

$$= \sqrt{225^2 + (-300)^2 + 450^2}$$

$$\boxed{\vec{F} = 585.77 \text{ N}}$$ Ans.

$$\theta_x = \cos^{-1}\left(\frac{F_x}{F}\right) = \frac{225}{585.77} = \boxed{67.41^\circ}$$

$$\theta_y = \cos^{-1}\left(\frac{F_y}{F}\right) = \frac{-300}{585.77} = \boxed{120.8^\circ}$$

$$\theta_z = \cos^{-1}\left(\frac{F_z}{F}\right) = \frac{450}{585.77} = \boxed{39.8^\circ}$$ Answer

3) Express force F in terms of unit vectors $\vec{i}$, $\vec{j}$ and $\vec{k}$. The force (F) 32 KN that starts from a point A (1, 2, −1.5) and Passes through the point B (−3, 4, 2).

Given data :-

F = 32 KN

A (1, 2, −1.5)

B (−3, 4, 2)

Unit Vector $\vec{n}_F = \frac{\vec{AB}}{|\vec{AB}|}$

$$= \frac{(x_2 - x_1)\vec{i} + (y_2 - y_1)\vec{j} + (z_2 - z_1)\vec{k}}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$$

$$= \frac{(-3-1)\vec{i} + (4-2)\vec{j} + (2+1.5)\vec{k}}{\sqrt{(-3-1)^2 + (4-2)^2 + (2+1.5)^2}}$$

$$= \frac{1}{\sqrt{32.25}} (-4\vec{i} + 2\vec{j} + 3.5\vec{k})$$

Force $\vec{F} = F . \vec{nF}$

$F = 32$ KN

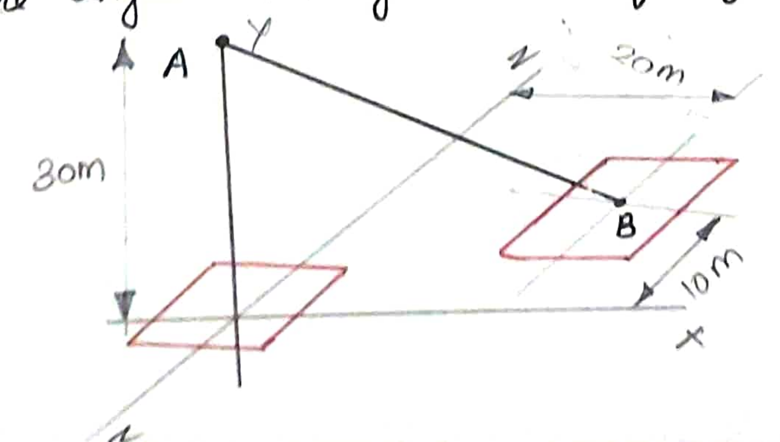
$$\vec{F} = 32 \times \frac{1}{\sqrt{32.25}} \times (-4\vec{i} + 2\vec{j} + 3.5\vec{k})$$

$$\boxed{\vec{F} = -22.53\vec{i} + 11.26\vec{j} + 19.71\vec{k}}$$ Answer.

4) A Vertical pole is guided by a wire AB which is anchored by means of a bolt at B as shown in figure. The tension in the wire is 1000 N. Determine

(i) The components of the force acting on the bolt and

(ii) The angles θ_x, θ_y & θ_z defining the direction of the force.



Solution :-

Point (A) position $A(x_1, y_1, z_1) = (0, 30, 0)$

" B " $B(x_2\, y_2\, z_2) = (20, 0, -10)$

$$\text{Vector } \vec{BA} = \vec{OA} - \vec{OB}$$

$$= (x_1 - x_2)\vec{i} + (y_1 - y_2)\vec{j} + (z_1 - z_2)\vec{k}$$

$$= (0-20)\vec{i} + (30-0)\vec{j} + (0-(-10))\vec{k}$$

$$\boxed{\vec{BA} = -20\vec{i} + 30\vec{j} + 10\vec{k}}$$

To find Force Vector $\vec{F}$

$$\vec{F} = F.\vec{n}_F$$

$$\vec{n}_F = \frac{\vec{BA}}{|\vec{BA}|}$$

$$|\vec{BA}| = \sqrt{(-20)^2 + (30)^2 + (10)^2} = 37.42$$

$$\vec{n}_F = \frac{-20\vec{i} + 30\vec{j} + 10\vec{k}}{37.42}$$

$$\vec{F} = F.\vec{n}_F$$

$$= 1000\left[\frac{-20\vec{i} + 30\vec{j} + 10\vec{k}}{37.42}\right]$$

$$\boxed{\vec{F} = -534.47\vec{i} + 801.7\vec{j} + 267.2\vec{k}}$$

$$\boxed{\begin{aligned} F_x &= -534.47 \text{ N} \\ F_y &= 801.70 \text{ N} \\ F_z &= 267.20 \text{ N} \end{aligned}}$$ Answers

Directions :-

$$\cos\theta_x = \frac{F_x}{F} = \frac{-534.7}{1000} = -0.534$$

$$\boxed{\theta_x = 122.31^\circ}$$

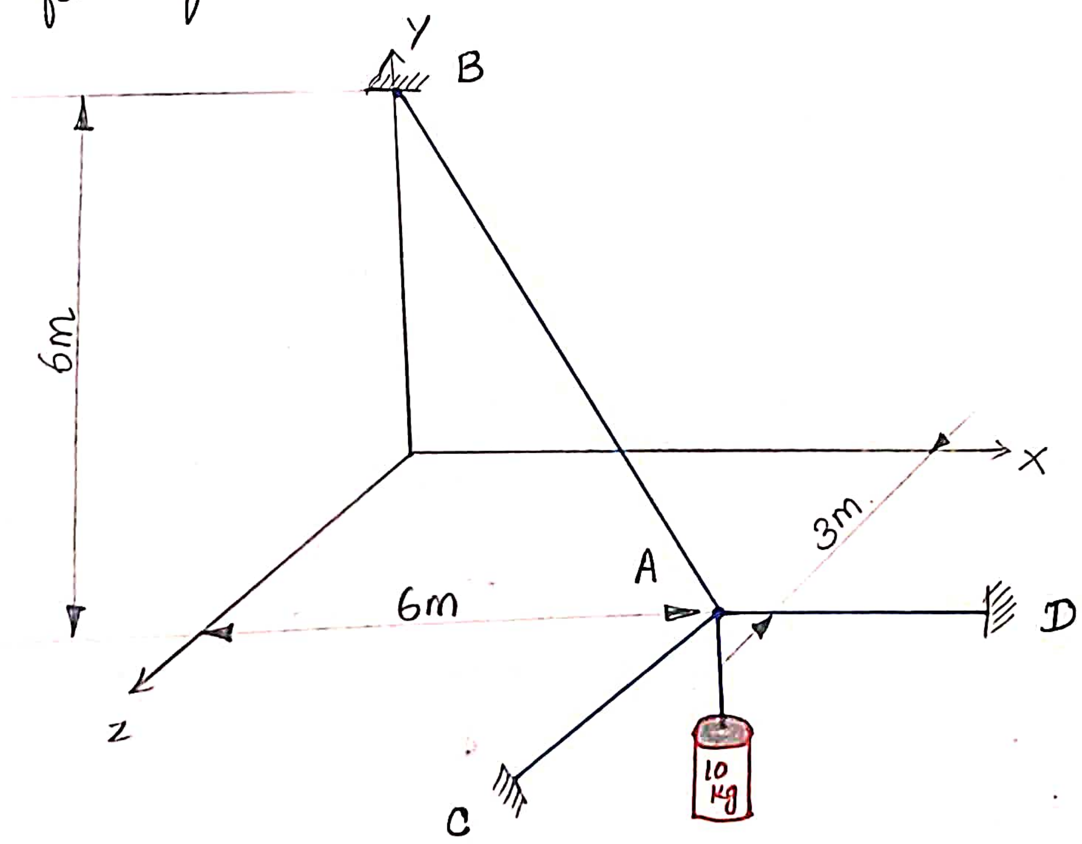
$$\cos\theta_y = \frac{F_y}{F} = \frac{801.7}{1000} = 0.8017$$

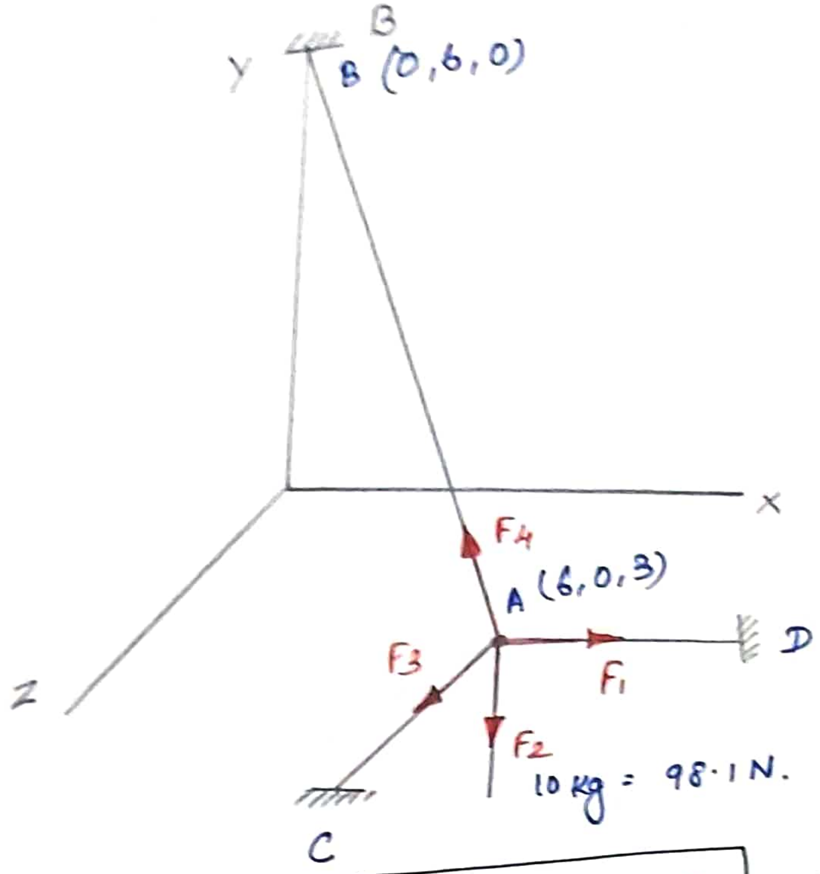
$$\boxed{\theta_y = 36.71^\circ}$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{267.2}{1000} = 0.2672$$

$$\boxed{\theta_z = 74.50^\circ}$$ Answers.

5) Three cables are used to support the 10 kg cylinder shown in figure. Determine the force developed in each cable for equilibrium.





For Equilibrium $\boxed{F_1 + F_2 + F_3 + F_4 = 0}$ ——— (1)

$\vec{F_1} = F_1 \vec{i}$; $F_2 = -98.1 \vec{j}$; $F_3 = F_3 \vec{k}$; $F_4 = F . \vec{n}_{AB}$

To find unit Vector :-

$$\vec{n}_{AB} = \frac{\vec{AB}}{|\vec{AB}|}$$

$$\vec{AB} = xi + yj + zk = (x_b - x_a)\vec{i} + (y_b - y_a)\vec{j} + (z_b - z_a)\vec{k}$$

$$= (0-6)\vec{i} + (6-0)\vec{j} + (0-3)\vec{k}$$

$$= -6\vec{i} + 6\vec{j} - 3\vec{k}$$

$$|\vec{AB}| = \sqrt{(-6)^2 + (6)^2 + (-3)^2} = 9 \text{ m}$$

$$\vec{n}_{AB} = \frac{-6\vec{i} + 6\vec{j} - 3\vec{k}}{9} = -0.67\vec{i} + 0.67\vec{j} - 0.33\vec{k}$$

To find F_4 :-

$$\vec{F_4} = F_4 \cdot \vec{n}_{AB}$$

$$\vec{F_4} = F_4 \left[-0.67\vec{i} + 0.67\vec{j} - 0.33\vec{k}\right]$$

To find values of F_1, F_3 & F_4 :-

Apply the values in equation ①

$$F_1\vec{i} - 98.1\vec{j} + F_3\vec{k} + F_4\left[-0.67\vec{i} + 0.67\vec{j} - 0.33\vec{k}\right] = 0$$

Equating the co-efficients of $\vec{i}$, $\vec{j}$ & $\vec{k}$ components to zero

$$F_1 - 0.67 F_4 = 0 \quad \text{——— ②}$$

$$-98.1 + 0.67 F_4 = 0 \quad \text{——— ③}$$

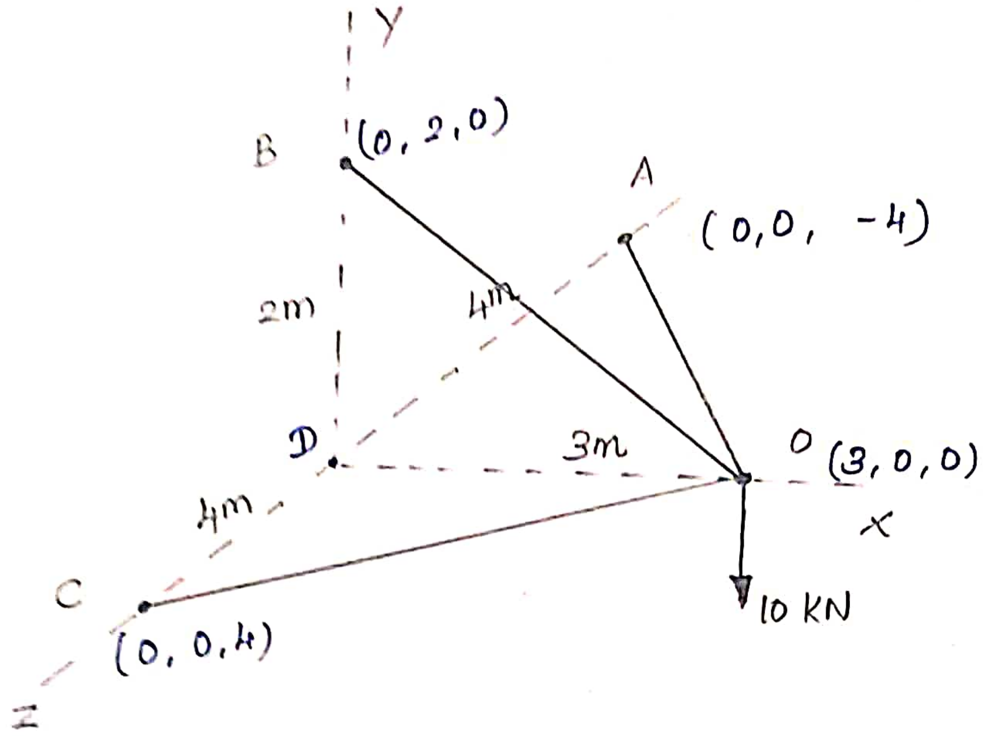
$$F_3 - 0.33 F_4 = 0 \quad \text{——— ④}$$

from eqn ③ , $\boxed{F_4 = 146.42\ N}$

Sub F_4 in eqn ④ $\boxed{F_3 = 48.32\ N}$

from eqn ② $\boxed{F_1 = 98.1\ N}$ Answers.

6) Members OA and OB and OC form a three member space truss. A weight of 10 kN is suspended at the joint O. Determine the magnitude and nature of forces induced in each of the three members of the truss.



Solution :-

Forces in member OA

$$\vec{OA} = \vec{DA} - \vec{DO} = (0\vec{i} + 0\vec{j} - 4\vec{K}) - (3\vec{i} + 0\vec{j} + 0\vec{k})$$

$$\vec{OA} = -3\vec{i} - 4\vec{K}$$

$$|\vec{OA}| = OA = \sqrt{3^2+4^2} = \sqrt{25} = 5$$

$$\vec{F_1} = F_1 \times \frac{\vec{OA}}{|OA|} = F_1 \times \left[\frac{-3\vec{i} - 4\vec{k}}{5}\right]$$

Forces in member OB

$$\vec{OB} = \vec{DB} - \vec{DO} = (0\vec{i} + 2\vec{j} + 0\vec{K}) - (3\vec{i} + 0\vec{j} + 0\vec{k})$$

$$\vec{OB} = -3\vec{i} + 2\vec{j}$$

$$|\vec{OB}| = \sqrt{3^2+2^2} = \sqrt{13}$$

$$\vec{F_2} = F_2 \times \frac{\vec{OB}}{|\vec{OB}|} = F_2 \times \left(\frac{-3\vec{i} + 2\vec{j}}{\sqrt{13}}\right)$$

(62)

Forces in member OC

$$\vec{OC} = \vec{DC} - \vec{DO} = (0\vec{i} + 0\vec{j} + 4\vec{k}) - (3\vec{i} + 0\vec{j} + 0\vec{k})$$

$$\vec{OC} = -3\vec{i} + 4\vec{k}$$

$$|\vec{OC}| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$$

$$\vec{F_3} = F_3 \times \frac{\vec{OC}}{|\vec{OC}|} = F_3 \times \left(\frac{-3\vec{i} + 4\vec{k}}{5}\right)$$

Force throught weight $F_4 = -10\vec{j}$

Equilibrium condition $\vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_4} = 0$

$$\left(\frac{-3\vec{i} - 4\vec{k}}{5}\right) F_1 + \left(\frac{-3\vec{i} + 2\vec{j}}{\sqrt{13}}\right) F_2 + \left(\frac{-3\vec{i} + 4\vec{k}}{5}\right) F_3 - 10\vec{j} = 0$$

Equaling the co-efficient of i, j & k

$$\frac{-3}{5} F_1 - \frac{-3}{\sqrt{13}} F_2 - \frac{-3}{5} F_3 = 0 \quad \text{------ (1)}$$

$$\frac{2}{\sqrt{13}} F_2 - 10 = 0 \quad \text{------ (2)}$$

$$-\frac{4}{5} F_1 + \frac{4}{5} F_3 = 0 \quad \text{------ (3)}$$

$$\therefore F_1 = F_3$$

using eqn (2) $\Rightarrow$ $\boxed{F_2 = 18.02 kN}$ Answer.

using the value of F_2 in eqn (1) we get

$$-0.6 F_1 - \frac{3}{\sqrt{13}} (18.02) - 0.6 F_1 = 0$$

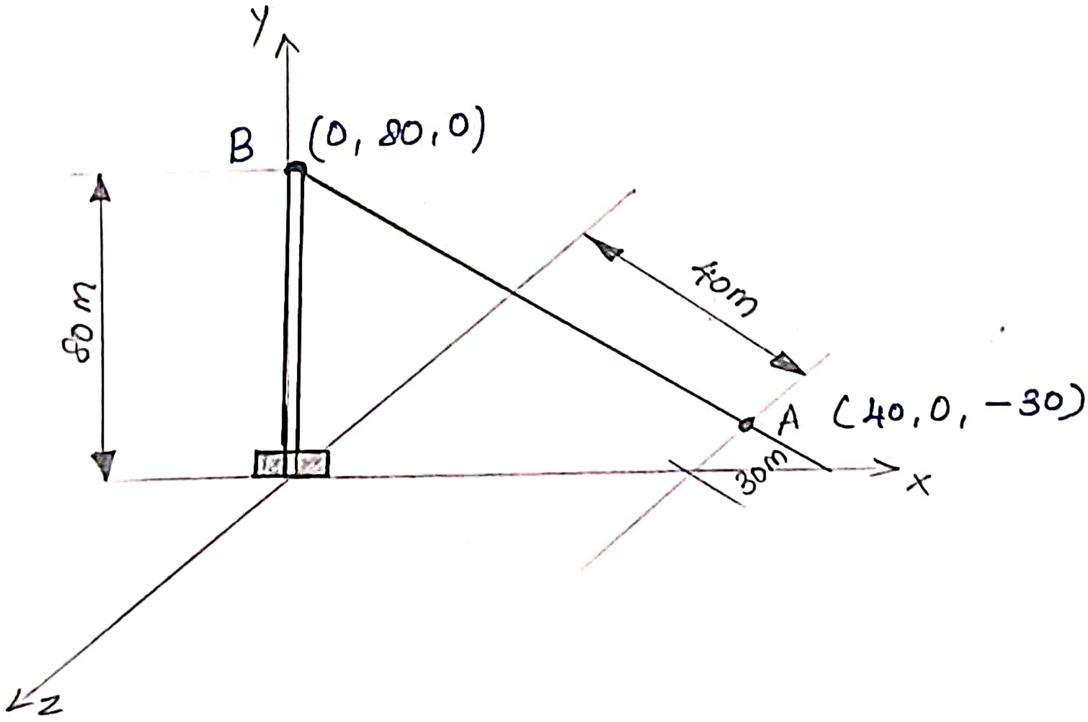
$$-1.2 F_1 = 14.99$$

$$\boxed{F_1 = -12.49 \text{ KN} = F_3}$$ Answer

7) A tower guy wire shown in figure is anchored by means of a bolt at A. This tension in the wire is 2500 kN. Determine

(i) The components F_x, F_y and F_z of the force acting on the bolt and.

(ii) The angle θ_x, θ_y and θ_z defining the direction of the forces.



Solution :-

Let AB the guy wire.

$$\vec{AB} = \vec{OB} - \vec{OA}$$

$$= (0-40)\vec{i} + (80-0)\vec{j} + (0-(-30))\vec{k}$$

$$\vec{AB} = -40\vec{i} + 80\vec{j} + 30\vec{k}$$

$$|\vec{AB}| = \sqrt{(-40)^2 + (80)^2 + (30)^2} = 94.34$$

$$\vec{F} = F \times \frac{\vec{AB}}{|\vec{AB}|}$$

$$= 2500\left[\frac{-40\vec{i} + 80\vec{j} + 30\vec{k}}{94.34}\right]$$

$$\vec{F} = \frac{2500}{94.34}\left[-40\vec{i} + 80\vec{j} + 30\vec{k}\right]$$

$$\boxed{\vec{F} = -1059.9\,\vec{i} + 2119.9\,\vec{j} + 794.9\,\vec{k}}$$

$$\boxed{\begin{aligned} F_x &= -1059.9 \text{ kN} \\ F_y &= 2119.9 \text{ kN} \\ F_z &= 794.9 \text{ kN} \end{aligned}}$$

Answer.

To fend angles :-

$$\cos\theta_x = \frac{F_x}{F} = \frac{-1059.9}{2500} \Rightarrow \theta_x = 115^\circ$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{2119.9}{2500} \Rightarrow \theta_y = 31.99^\circ$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{794.9}{2499.6} \Rightarrow \theta_z = 71.46^\circ$$

Ans

UNIT - II

EQUILIBRIUM OF RIGID BODIES

Syllabus :

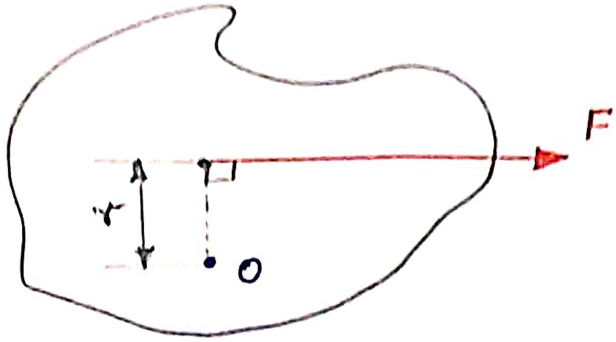
Principle of Transmissibility, Equivalant forces, Vector product of Two Vectors, Moment of a force about a point, Varignon's theorem, Rectangular components of the moment of a force, Scalar product of Two vectors, Mixed Triple product of three vectors, Moment of a force about an axis, Couple - Moment of a couple, Equivalant couples, Addition of Couples, Resolution of a given Force into a force - Couple system, Further reduction of a system of forces, Equilibrium in two and three dimensions - Reactions at supports and connections.

Principle of Transmissibility :-

It states that, the state of rest or motion of a rigidi body is unaltered, if a force acting on a body is replaced by another force of same magnitude and direction, but acting anywhere on the body along the line of action of the replaced force.

Moment of a force about a point :-

It can be defined as, the product of magnitude of force and the perpendicular distance between the force (line of action) and the point (about which moment has to be taken).



The moment (M) of the force 'F' about 'O' is given by

$$M = \vec{F} \times \vec{r}$$

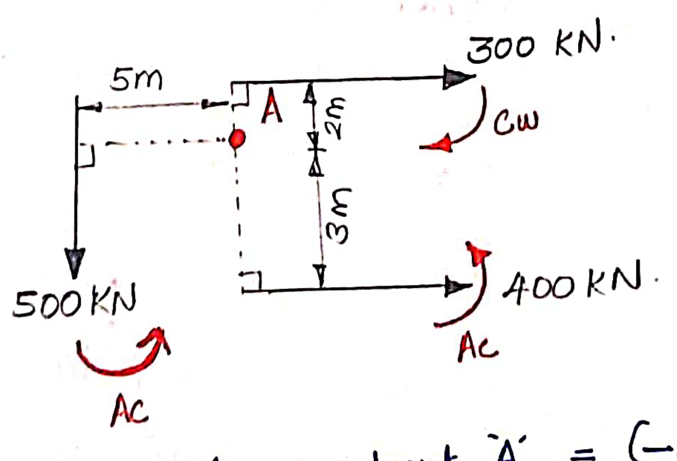
$\vec{F}$ = A force acting on the body

$\vec{r}$ = Perpendicular distance from the point 'O' on the line of action of force 'F'.

Clockwise moment ↻ Positive

Anticlockwise moment ↺ Negative.

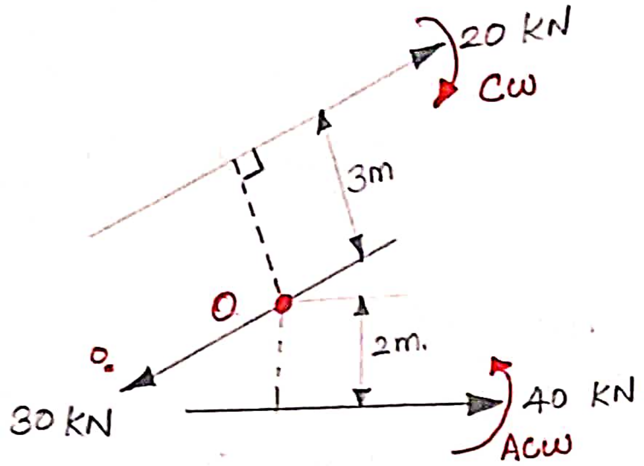
1) Find the moment of the force about A, if the forces acting are 300 KN, 400 KN and 500 KN as shown in figure.



Moment of a force about 'A' $= (-500 \times 5) + (-400 \times 3) + (300 \times 2)$

$= -3100$ KN.m (anti clockwise)

2) When a system of forces act on the body as shown in figure. Find the moments about 'O'.



Moment about 'O' $= \Sigma \vec{r} \times \vec{F}$

$= (30 \times 0) + (-40 \times 2) + (20 \times 3)$

$= -20$ KN·m.

VARIGNON'S THEOREM

Principle of Moments :-

Varignon's theorem states that, the momentum of the resultant of number of forces about any point is equal to the algebraic sum of the moments of all the forces of the system about the same point.

$\vec{M} = \vec{r} \times \vec{R}$

$\vec{M}$ = The moment of the resultant about any point

$\vec{R}$ = Resultant of the given forces

$\vec{r}$ = Distance between the forces

$\vec{M} = \vec{r} \times (\vec{F_1} + \vec{F_2} + \vec{F_3} + \dots + \vec{F_n})$

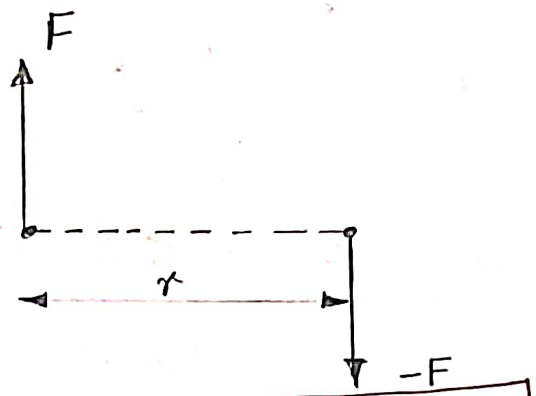
$\vec{R} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_4} \dots + \vec{F_n}$

$\vec{M} = (\vec{r_1} \times \vec{F_1}) + (\vec{r_2} \times \vec{F_2}) + (\vec{r_3} \times \vec{F_3}) + \dots + (\vec{r_n} \times \vec{F_n})$

Moment of a Couple :-

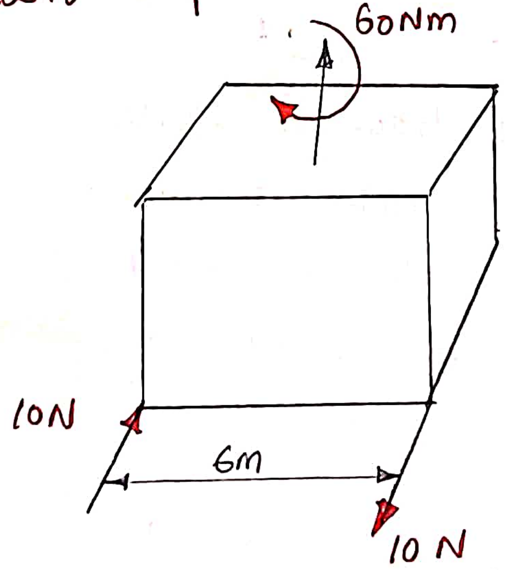
Couple :-

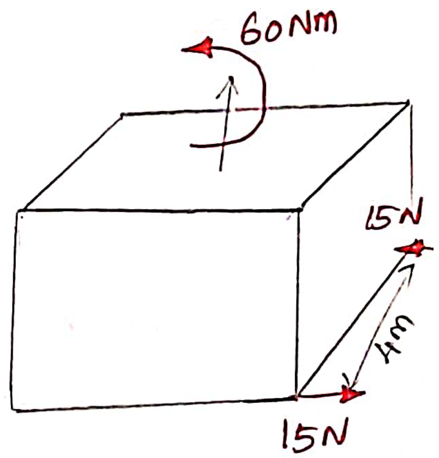
The moment produced by two equal, opposite and non collinear force is known as a couple.



Moment of a couple $\boxed{\vec{M} = \vec{r} \times \vec{F}}$

Equivalant couple :-





Addition of couple :-

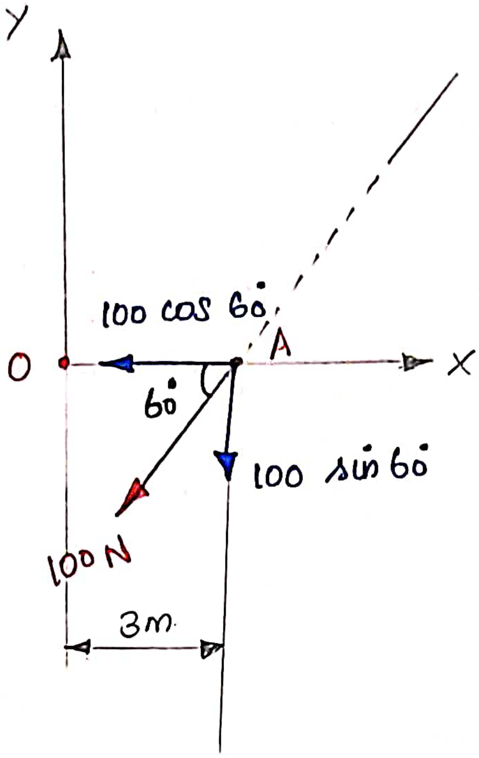
$$\vec{M_1} = \vec{F_1} \times \vec{r_1}$$

$$\vec{M_2} = \vec{F_2} \times \vec{r}$$

$$\therefore M = (\vec{F_1} + \vec{F_2}) \vec{r}$$

$$\boxed{\vec{N} = \vec{M_1} + \vec{M_2}}$$

1) A force of 100 N is acting at a point A shown in figure. Determine the moments of this force about O.



Solution :-

Horizontal component of the force = 100 cos 60°

this force component passing through the 'O' and no moment.

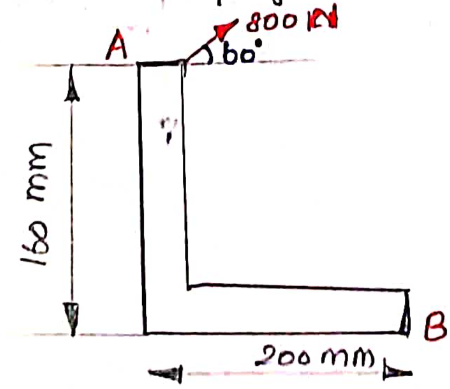
Vertical component = 100 sin 60° = 86.66 N.

Moment of this force = 86.66 × 3 = 259.8 N.m

$\boxed{M = 259.8 \text{ Nm}}$ Clockwise

Answer.

2) Determine the moment of force 800 N acting on a bracket about 'B'



The moment of force about 'B'

= (Horizontal component of force × Perpendicular distance)

+ (Vertical component of force × Perpendicular distance)

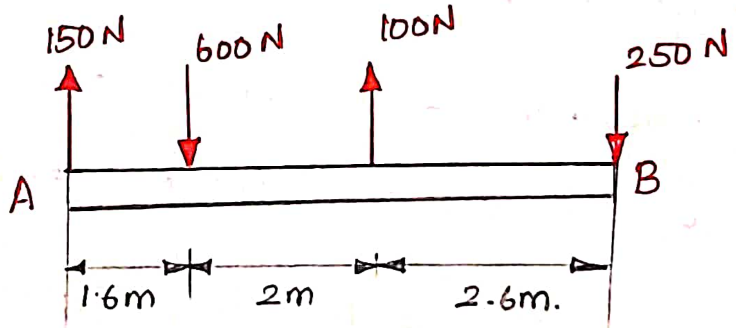
$= (800 \cos 60^\circ \times 160) + (800 \sin 60^\circ \times 200)$

$= 202564.04$ Nmm.

$\boxed{M_B = 202.564 \text{ Nm.}}$ ∴ Clockwise.

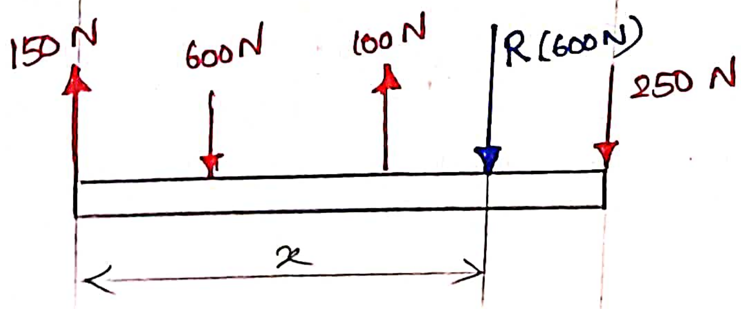
Answer.

3. 6.2m beam is subjected to the forces shown in figure. Reduce the system of forces to a single force (or) resultant and the distance of the resultant from point 'A'.



Resultant $R = \Sigma F = 150 - 600 + 100 - 250$

$R. = -600 N \; (\downarrow)$



By using varignon's theorem

Taking moment about 'A'

$$R \times x = M_A.$$

$$(600 \times x) = 600 \times 1.6 - 100 \times 3.6 + 250 \times 5.8$$

$$600x = -2050$$

$$\boxed{x = -3.42\ m}$$

Resultant ~~of~~ is at a distance of 3.42 m from A.

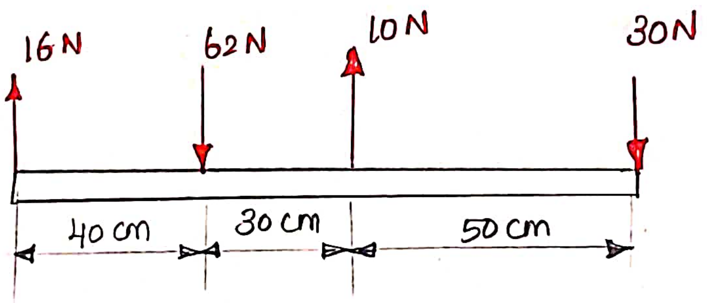
---

4) A rigid bar is subjected to a system of parallal forces as shown in figure. Reduce this system to

(i) A single force

(ii) A single force and moment at A

(iii) A single force moment system at B



Solution :-

(i) Single force (Resultant force)

$$\Sigma F \Rightarrow 16 - 62 + 10 - 30$$

$$\boxed{R = -66\ N}$$

(ii) Single force moment system at A

$$M_A = 62 \times 40 - 10 \times (40+30) + 30 \times (40+30+50)$$

$$\boxed{M_A = 5380\ N\,cm \ \text{(Clockwise)}}$$

(iii) Single force moment at 'B'

$$M_B = 16 \times 120 - 62 \times 80 + 10 \times 50$$

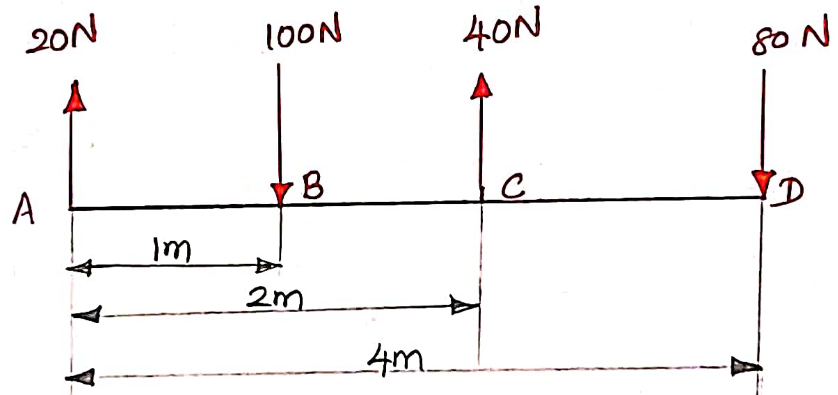
$$\boxed{M_B = -2540 \text{ Ncm (Anti clock wise)}}$$

5) A system of forces are acting on rigid bar as shown in figure. Reduce this system to

(i) A single force

(ii) A single force and a couple at A

(iii) A single force and a couple at B



A single force :-

$$\text{Resultant force } \Sigma F_y = 20 - 100 + 40 - 80$$

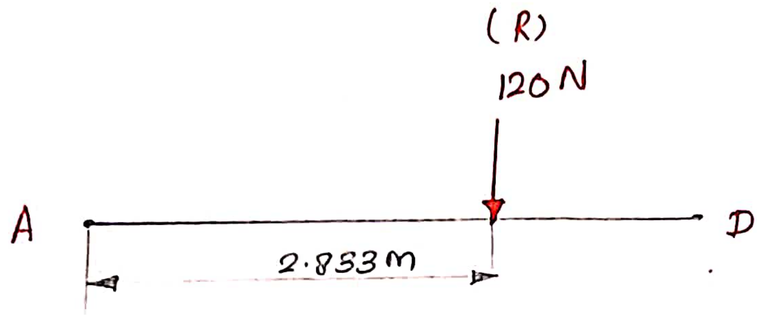
$$= -120 \text{ N}$$

Distance of resultant force from 'A'

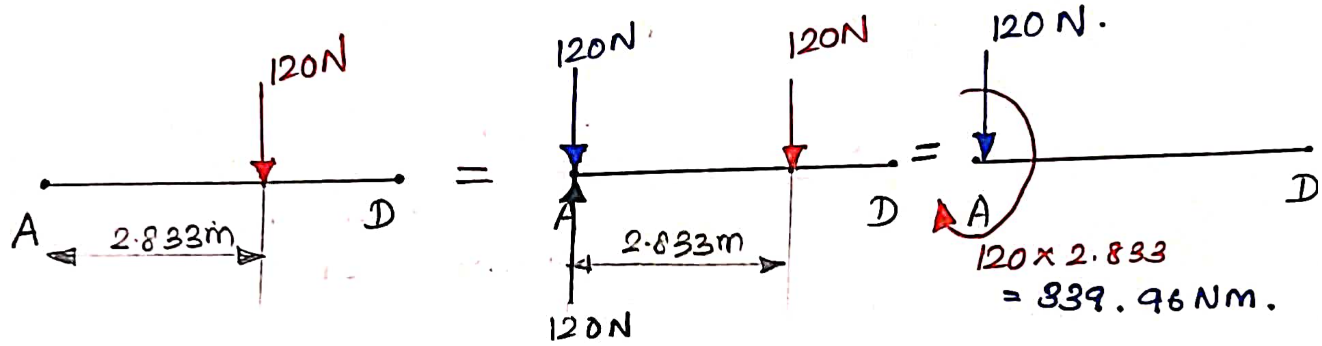
$$R \times x = M_A$$

$$120 \times x = 100 \times 1 - 40 \times 2 + 80 \times 4$$

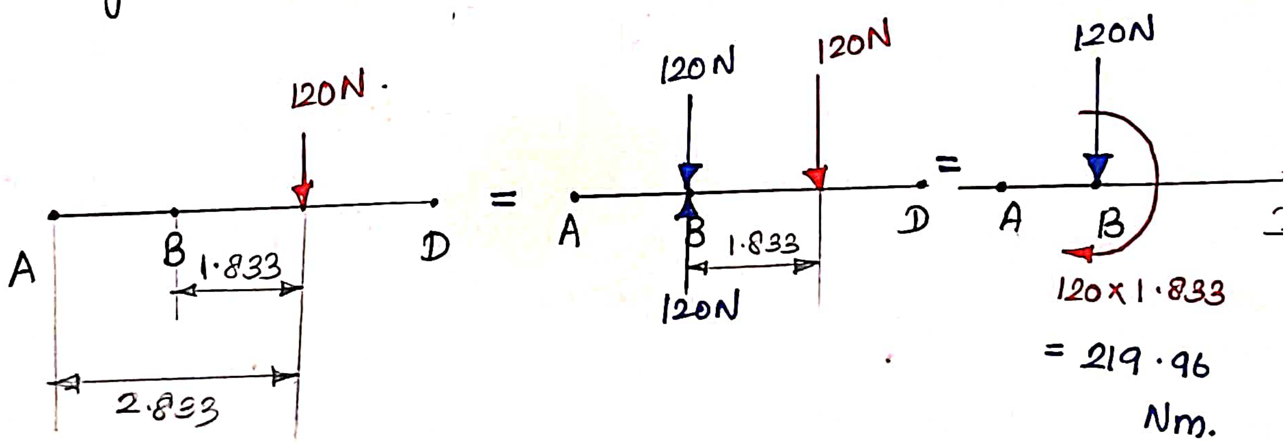
$$x = \frac{340}{120} = 2.833$$



(ii) Single force and couple at 'A'



(iii) Single force and couple at D



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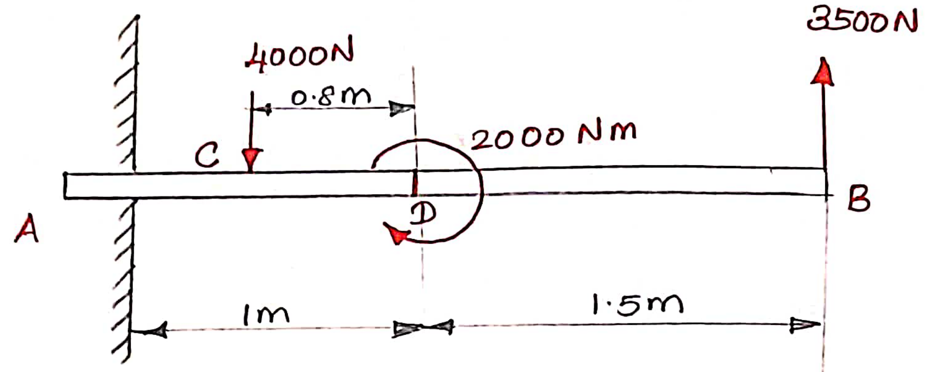
(6) Figure shows, two vertical forces and a couple of moment 2000 Nm acting on a horizontal rod which is fixed at end 'A'.

(i) Determine the resultant of the system

(ii) Determine an equivalant system through A.



(i) Resultant of the system :-

Resultant force

$R = 3500 - 4000$

$R = -500$ N.

$M_a = R \times x.$

$M_a \Rightarrow$ Moment about 'A'

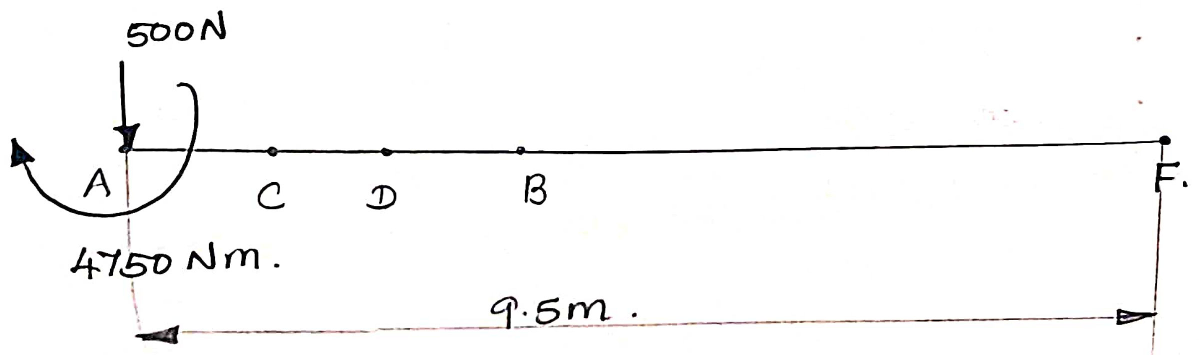
$(4000 \times 0.2) + 2000 - (3500 \times 2.5) = R \times x.$

$-4750 = 500 \times x$

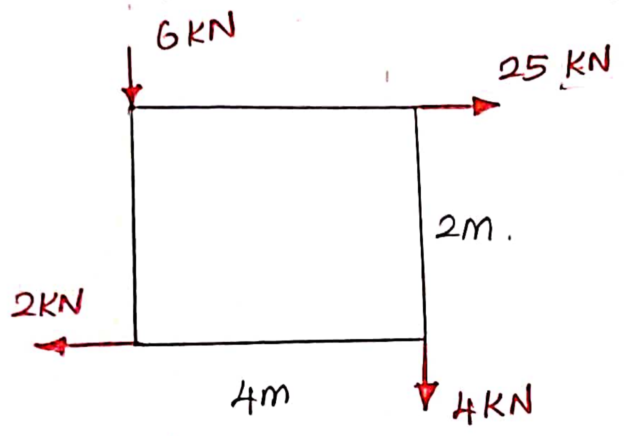
$x = -9.5$ m.

Resultant force acting 9.5 m from 'A' (downward) (500 N)

(ii) Equivalent system through 'A'



7) Determine the magnitude and direction of a single force which keeps the system in equilibrium. The system of forces acting is shown in figure.



Solution:-

Magnitude of Resultant $R = \sqrt{\Sigma H^2 + \Sigma V^2}$

$\Sigma H = 25\ KN - 2\ KN = 23\ KN$

$\Sigma V = -6\ KN - 4\ KN = -10\ KN.$

$R = \sqrt{23^2 + (-10)^2} = \boxed{25.07\ KN}$ Ans.

Angle of resultant with horizontal $\left(\frac{\Sigma V}{\Sigma H}\right)$

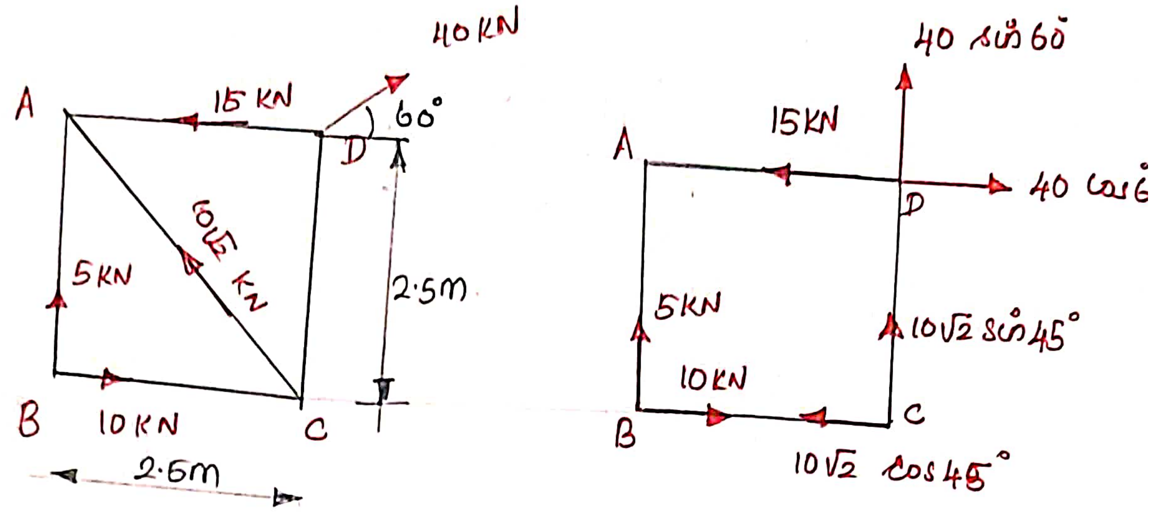
$\tan \alpha = \frac{-10}{23}$

$\alpha = \tan^{-1}\left(\frac{-10}{23}\right) = -23.49^\circ$

$\alpha = 360 - 23.49^\circ$

$\boxed{\alpha = 336.50^\circ}$ Ans.

8) Calculate the resultant moment about the corner B' as shown in figure.



Taking moment about point B

$$\Sigma M_B = (40 \cos 60^\circ \times 2.5) - (40 \sin 60^\circ \times 2.5) - (15 \times 2.5)$$
$$- (10\sqrt{2} \sin 45^\circ \times 2.5)$$

$$\boxed{= -99.10 \text{ KNm.}}$$ Answer.

Anticlockwise.

EQUILIBRIUM OF RIGID BODY IN TWO DIMENSIONS.

When a stationary body is acted upon by some external forces, the body may start to rotate or may start to move about any point. If the body does not move or rotate about any point then, the body is said to be in equilibrium.

Principle of Equilibrium :

$$\Sigma F = 0 \Rightarrow \Sigma H = 0 \quad \& \; \Sigma V = 0$$

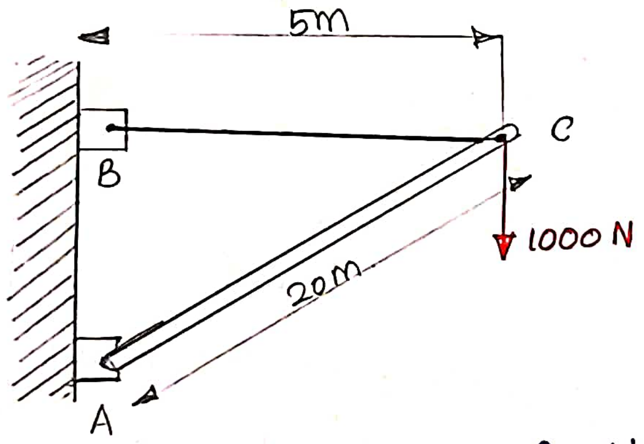
$$\Sigma M = 0 \Rightarrow \Sigma M = 0$$

Moment law of Equilibrium :-

When the body is in equilibrium state, then the summation of moments of forces acting on the body is 'zero'.

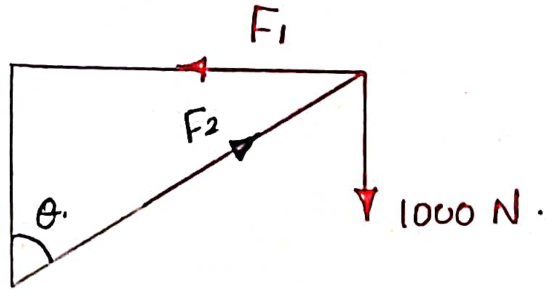
---

1) A beam 20 m long supports a load of 1000 N as shown in figure. The cable BC is horizontal and 5 m long. Determine the force in the cable and the Beam.



$$\sin\theta = \frac{5}{20} = \frac{1}{4} \Rightarrow \theta = 14.47^\circ$$

$$AB = 20\cos\theta = 20\cos 14.47^\circ = 19.36 \text{ m.}$$



Taking moment of all the forces about 'A'

$$\Sigma M_A = 0 \Rightarrow (1000 \times 5) - P_1 \times 19.36 = 0$$

$$\boxed{F_1 = 258.26 \text{ N.}}$$ Answer.

$$\Sigma V = 0 \Rightarrow F_2 \cos\theta - 1000 = 0.$$

$$F_2 \cos 14.47^\circ - 1000 = 0$$

$$\boxed{F_2 = 1032.76 \text{ N}}$$ Answer.

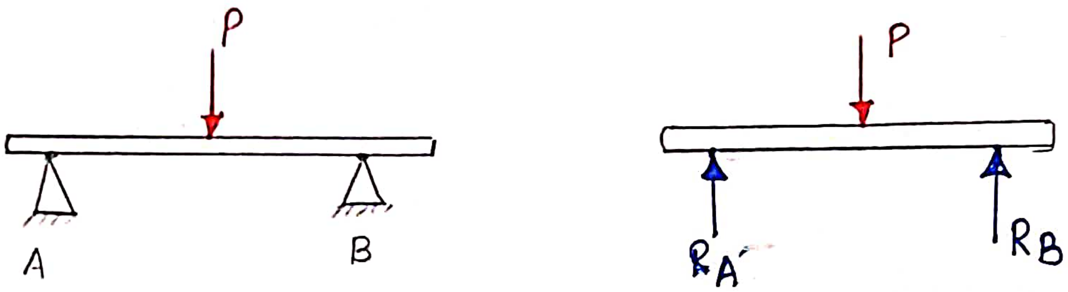
SUPPORTS

When a given body is supported by some external forces, the supporting forces will exert a force on the given body in order to make the body in equilibrium state.

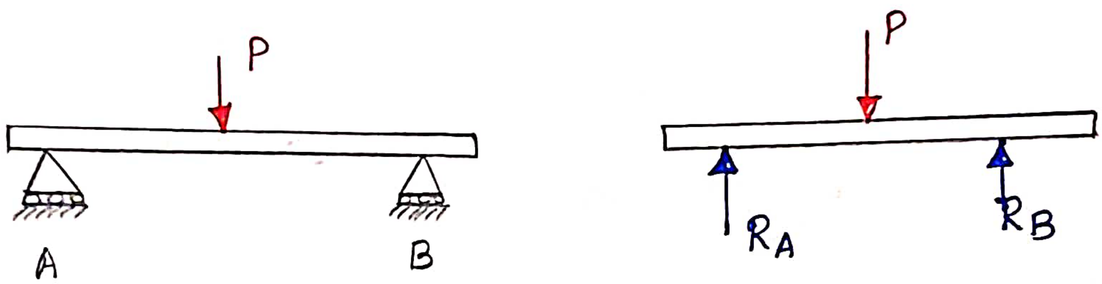
These bodies or forces are represented by reactions on corresponding points of contact with the given body. These bodies are called "supports".

Types of supports :-

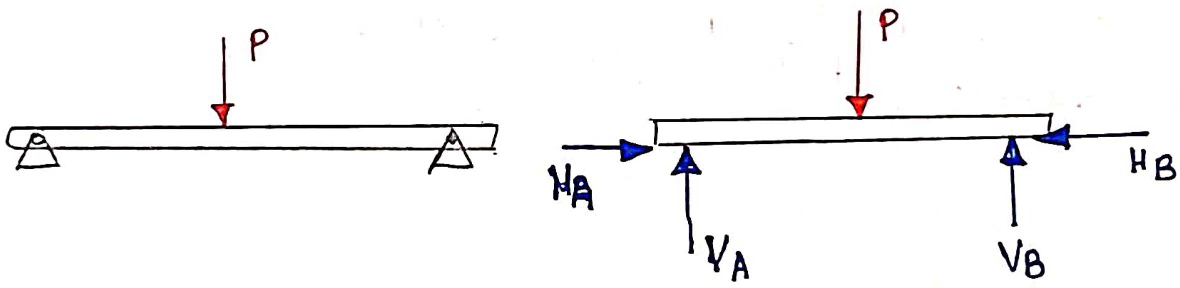
1. Simple supports (Knife edge supports)
2. Roller supports
3. Hinged support (pin joint)
4) Fixed support.



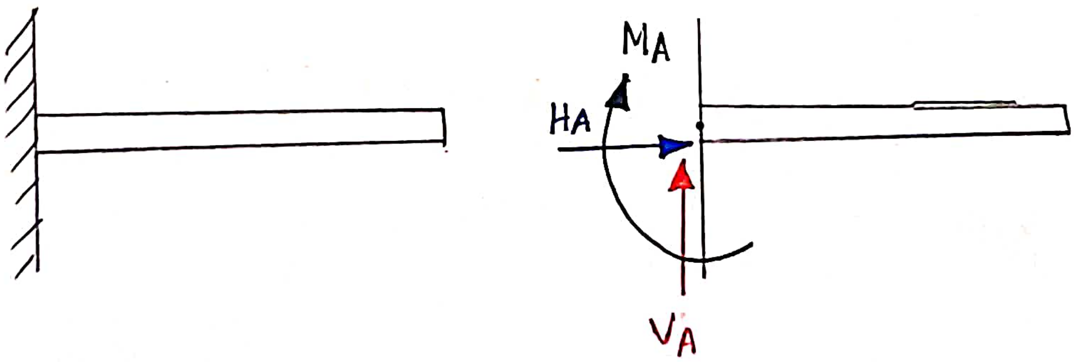
Simple supports



Roller support.



Hinged Support

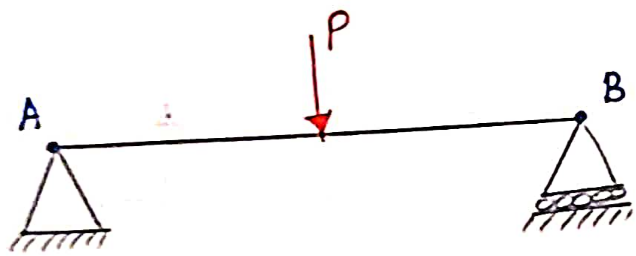


Fixed Support.

Types of Load :-

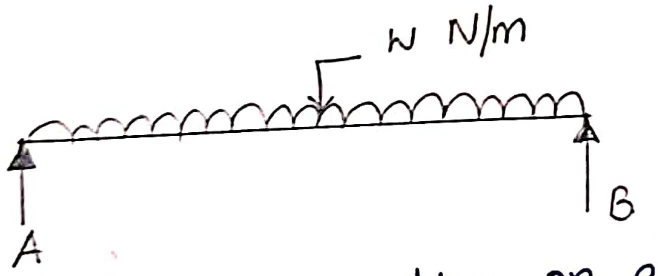
(i) Point load
(ii) uniformly Distributed Load (UDL)
(iii) uniformly Varying load. (UVL)

Point load :-



A Load, which is acting at a point on a beam is known as point load.

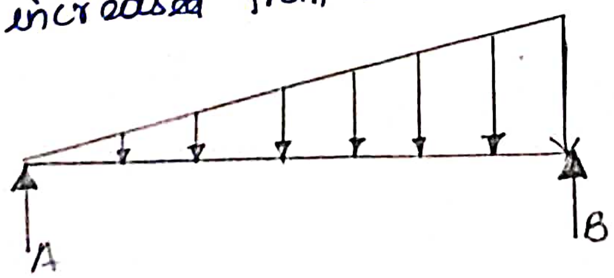
uniformly distributed load :-



A load which is acting on a beam for entire length.

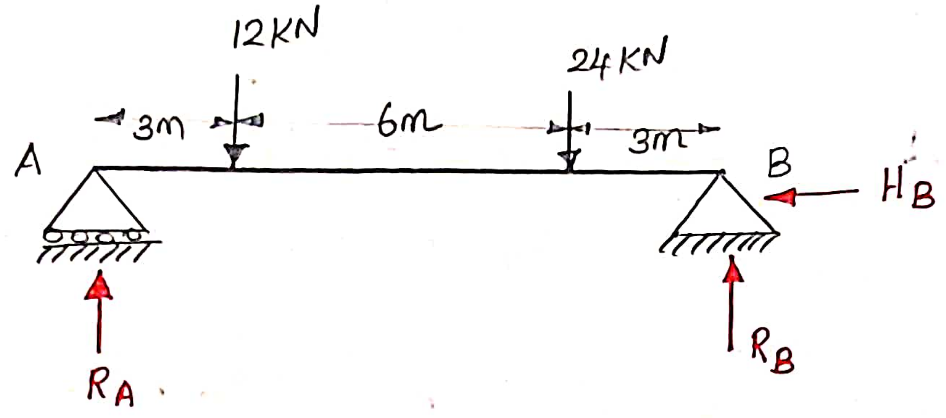
uniformly varying Load :

Load is increased from one end to other end gradually.



Problems on Reactions (Support Reactions on Beam)

1) Find the support reactions of a simply supported beam as shown in figure.



Solution :-

$\Sigma V = 0 \Rightarrow R_A + R_B - 12 - 24 = 0$

$R_A + R_B = 36$ —— ①

$\Sigma M_A = 0 \Rightarrow (12 \times 3) + (24 \times 9) - (R_B \times 12) = 0$

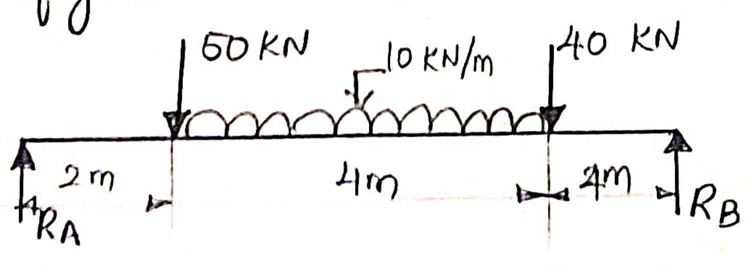
$R_B = \frac{252}{12} = 21$ KN.

$\boxed{R_B = 21 \text{ KN}}$ sub in eqn ①

$\boxed{R_A = 15 \text{ KN.}}$

$\Sigma H = 0 \Rightarrow \boxed{H_B = 0}$ Answers.

2) A simply supported beam of length 10m, carries the uniformly distributed load and two point loads as shown in figure. Calculate the reactions at R_A & R_B.



Total load due to UDL $= 4 \times 10 = 40$ KN

UDL of 40 KN will be acting at the middle point of CD i.e. at a distance of $4/2 = 2m$ from 'c' (or) at a distance of $2+2 = 4$ m from point 'A'.

$\Sigma V = 0 \Rightarrow R_A + R_B - 50 - (10 \times 4) - 40 = 0$

$R_A + R_B = 130$ ——— ①

$\Sigma M_A = 0 \Rightarrow (50 \times 2) + (40 \times 2 \times 4/2) + (40 \times 6) - (R_B \times 10) = 0$

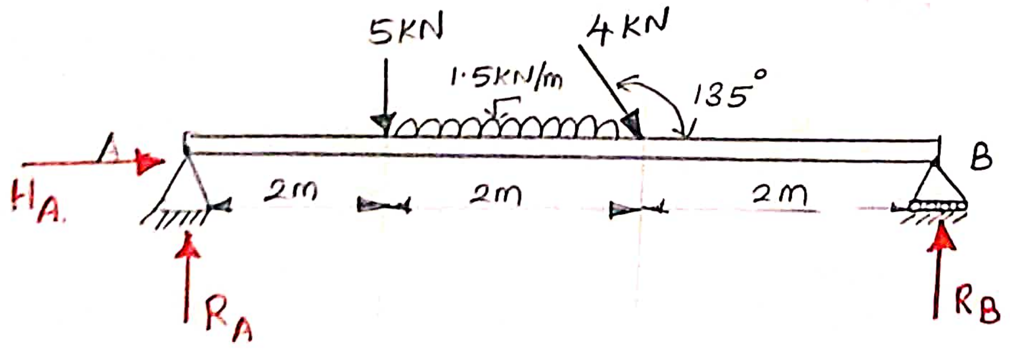
$10R_B - 240 - 160 - 100 = 0$

$10R_B = 500$

$\boxed{R_B = 50 \text{ KN}}$

from eqn ① $\Rightarrow$ $\boxed{R_A = 80 \text{ KN}}$ Answers.

3) A beam AB 6m long is loaded as shown in figure. Determine the support reactions at 'A' & 'B'



$\Sigma V = 0 \qquad R_A + R_B - 5 - (1.5 \times 2) - (4 \sin 45) = 0$

$R_A + R_B = 10.828 \quad \text{———} \quad (1)$

$\Sigma H = 0$

$H_A + 4 \cos 45° = 0$

$\boxed{H_A = -2.828 \text{ kN}}$

$\Sigma M_A = 0$

$(-R_B \times 6) + (5 \times 2) + \left[2 \times 1.5 \times (2 + 2/2)\right] + \left[4 \sin 45° \times (2+2)\right] = 0$

$\boxed{R_B = 5.052 \text{ kN}}$

Substitute in eqn (1) $\boxed{R_A = 5.776 \text{ kN.}}$

Reactions at A $\Rightarrow \sqrt{H_A^2 + R_A^2}$

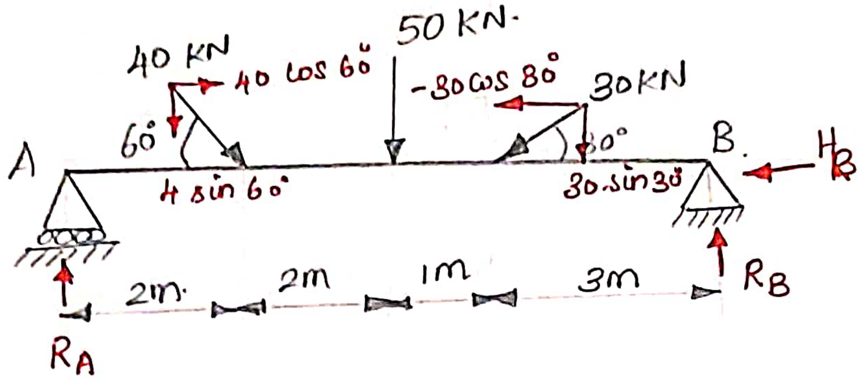
$= \sqrt{(-2.828)^2 + (5.776)^2}$

$\boxed{= 6.43 \text{ kN.}}$ Ans.

$\tan\theta = \frac{R_A}{H_A} = \frac{5.776}{2.828} = 2.0424$

$\boxed{\theta = 63.9°}$ Ans.

4. Find the reaction components of the beam shown in figure.



Solution :-

$\Sigma H = 0 \Rightarrow 40 \cos 60° - 30 \cos 30° - H_B = 0$

$$\boxed{H_B = -5.9807 \text{ N.}}$$ Ans.

$\Sigma V = 0 \Rightarrow R_A + R_B - 40 \sin 60° - 50 - 30 \sin 30 = 0$

$R_A + R_B = 99.64$ KN. ——— ①

$\Sigma M_A = 0 \Rightarrow (40 \sin 60° \times 2) + (50 \times 4) + (30 \sin 30° \times 5) - (R_B \times 8) = 0$

$8 R_B = 344.28$

$R_B = \frac{344.28}{8}$

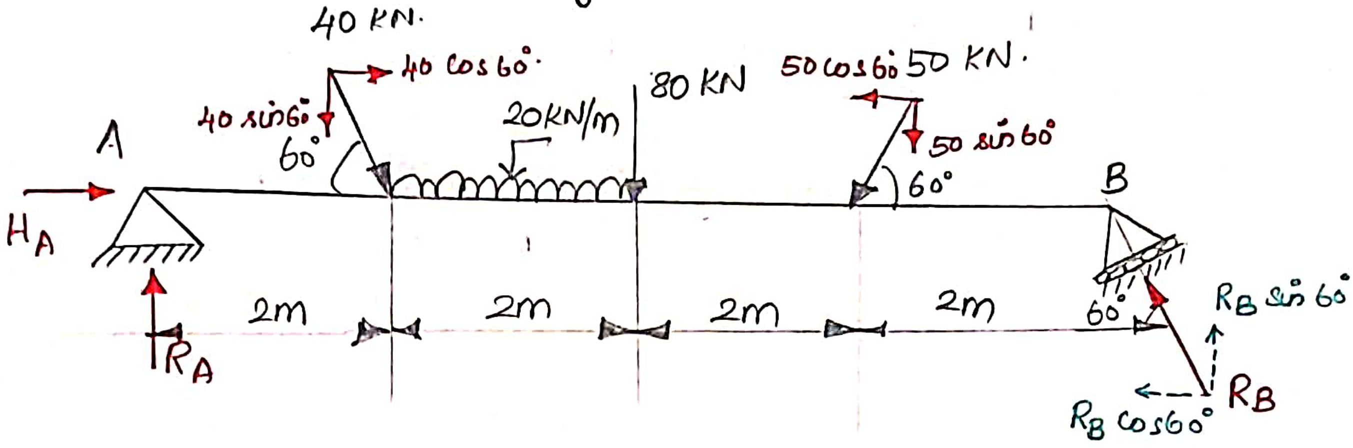
$$\boxed{R_B = 43 \text{ KN.}}$$ Ans.

Substitute in eqn ①

$R_A = 99.64 - 43$

$$\boxed{R_A = 56.64 \text{ KN}}$$ Ans

5) Find the reactions at supports 'A' and 'B' of the beam as shown in figure.



Solution:-

$\Sigma H = 0 \Rightarrow$

$$H_A + 40\cos 60° - 50\cos 60° - R_B \cos 60° = 0$$

$$H_A - 0.5 R_B = 5 \quad \text{———} \ (1).$$

$\Sigma V = 0 \Rightarrow$

$$R_A + R_B \sin 60° - 40 \sin 60° - 80 - (20 \times 2) \cancel{(2+2/2)} - 50 \sin 60° = 0.$$

$$R_A + R_B \sin 60° = 197.94$$

$$R_A + 0.866 R_B = 197.94 \quad \text{———} \ (2).$$

$\Sigma M_A = 0 \Rightarrow$

$$(40 \sin 60° \times 2) + (80 \times 4) + \left[(20\times2)\times(2+2/2)\right] + (50 \sin 60° \times 6) - (R_B \sin 60° \times 8) = 0.$$

$$8 R_B \sin 60° = 96.135 \text{ KN}.$$

$$\boxed{R_B = 111 \text{ KN}}$$ Ans.

Sub. in eqn ② $R_A = -60.5$ KN

from eqn ① $H_A = 101.81$ KN

Resultant at A $\Rightarrow \sqrt{R_A^2 + H_A^2}$

$= \sqrt{(-60.5)^2 + (101.81)^2}$

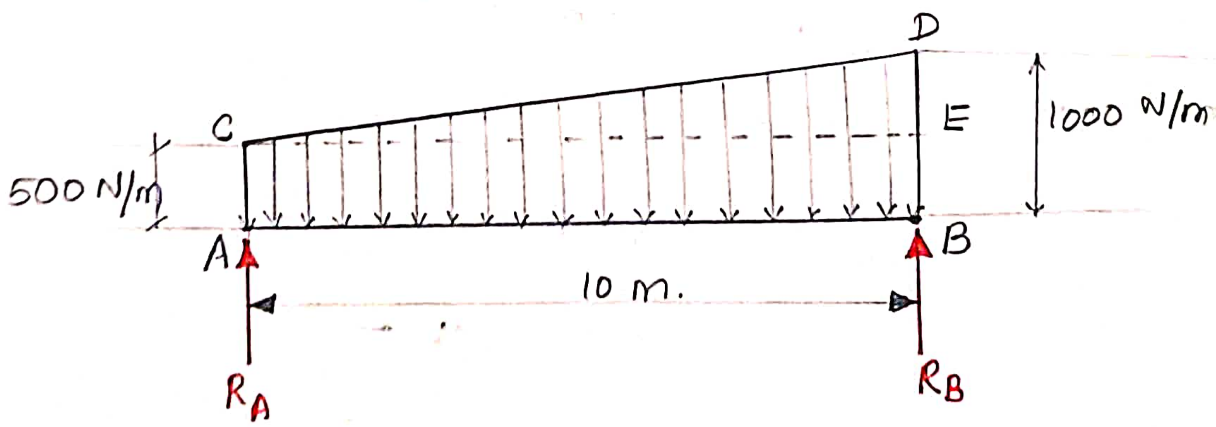
$= 118.43$ KN.

$\boxed{\text{Resultant at } A = 118.43 \text{ KN.}}$

$R_A = 101.81$ KN
$H_A = 60.5$ KN
$R_B = 118$ KN

Ans.

(b) A simply supported beam of length 10 m carries a uniformly increasing load of 500 N/m at one end to 1000 N/m at the other end. Calculate the reactions at both ends.



The C.G. of the rectangle ABEC will be at a distance of $10/2 = 5\,m$ from 'A'

The C.G of the triangle CED will be at a distance of $2/3 \times 10 = 6.67\,m$ from 'A'

$\Sigma F_y = \Sigma V = 0 \Rightarrow$

$R_A + R_B$ = Total load on the beam

$= (500 \times 10) + (\cancel{1/2} \times \cancel{10}^{5} \times 500)$

$= 5000 + 2500$

$R_A + R_B = 7500\ N.$ ——— (1)

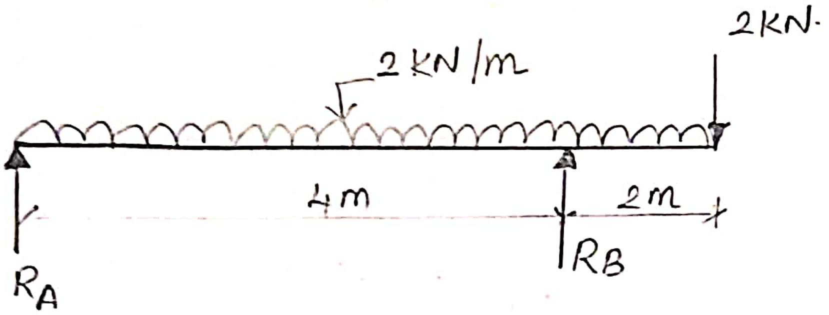
$\Sigma M_A = 0 \Rightarrow (R_B \times 10) - \left[(10 \times 500) \times 10/2\right] - \left[1/2 \times 10 \times 500 \times 2/3 \times 10\right] = 0$

$10\,R_B = 41666.67$

$\boxed{R_B = 4166.667\ N.}$

Sub in eqn (1) $\boxed{R_A = 3333.333\ N}$ Ans.

7) A beam AB of span 4m, overhanging on one side upto a length of 2m carries a uniformly distributed load of 2 kN/m over the entire length of 6m and a point load of 2 kN/m as shown in figure. Calculate the reactions at A and B.



Total load on beam due to UDL = 2×6 = 12 KN.

$$\Sigma V = 0 \Rightarrow R_A + R_B = 12 + 2 = 14 \quad \text{—— (1)}$$

$$\Sigma M_A = 0 \Rightarrow (R_B \times 4) - [(2\times6)\times3] - [2\times(4+2)] = 0$$

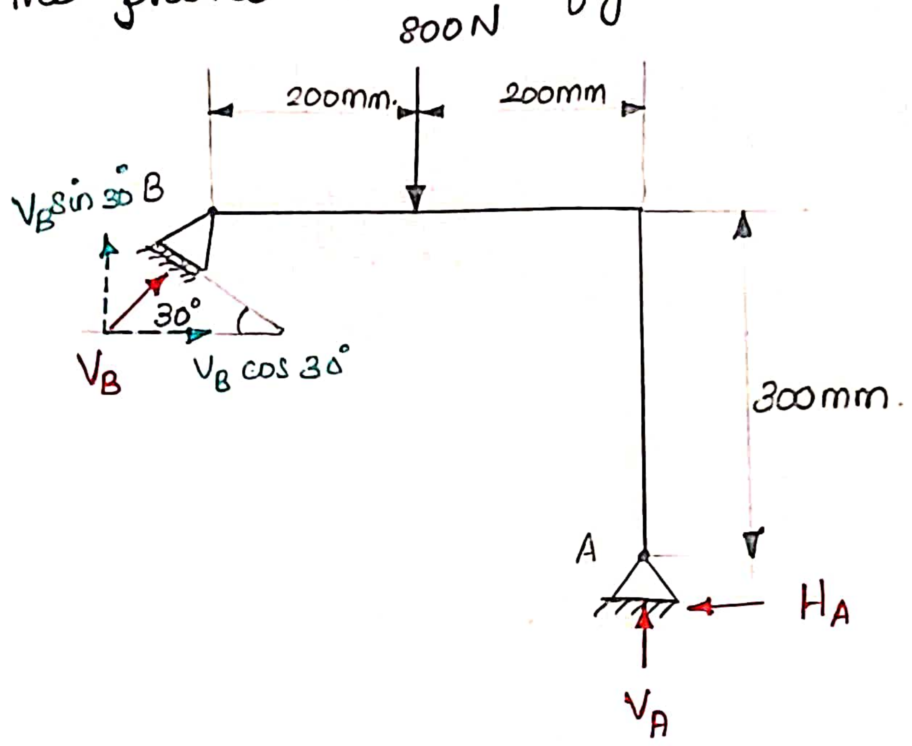
$$4R_B - 36 - 12 = 0$$

$$\boxed{R_B = 12\ KN} \quad \text{Ans.}$$

Sub. in eqn (1)

$$\boxed{R_A = 2\ KN.} \quad \text{Ans}$$

5) For the frame shown in figure determine the reactions at 'A'.



Solution :-

$\Sigma M_A = 0 \quad (V_B \cos 60° \times 300) + V_B (\sin 60° \times 400) - (800 \times 200) = 0$

$V_B \times 496.41 = 800 \times 200$

$\boxed{V_B = 322.31 \text{ N.}}$

$\Sigma H = 0 \Rightarrow \quad R_B \cos 60° - H_A = 0$

$\boxed{H_A = 161.15 \text{ N}}$

$\Sigma V = 0 \Rightarrow \quad N_B \sin 60° - 800 + V_A = 0$

$\boxed{V_A = 520.87 \text{ N.}}$

$R_A = \sqrt{(V_A)^2 + (H_A)^2}$

$= \sqrt{(520.87)^2 + (161.15)^2}$

$\left.\begin{matrix} V_A \\ H_A \end{matrix}\right\} \Rightarrow$ Resultant at $\boxed{A \Rightarrow 545.22\ N.}$

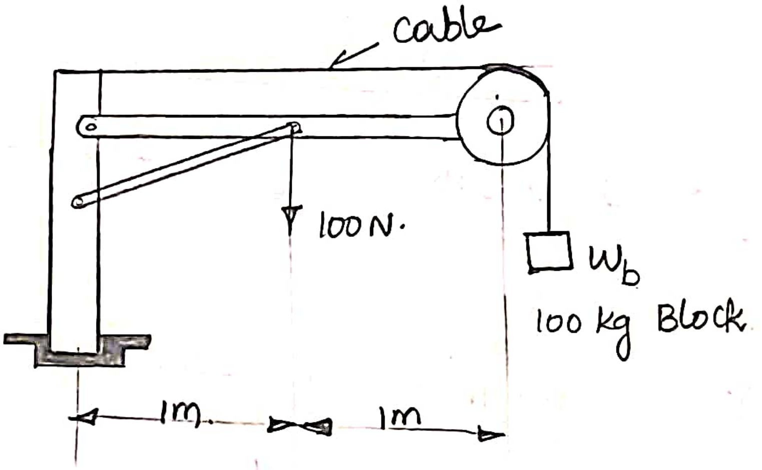
Ans

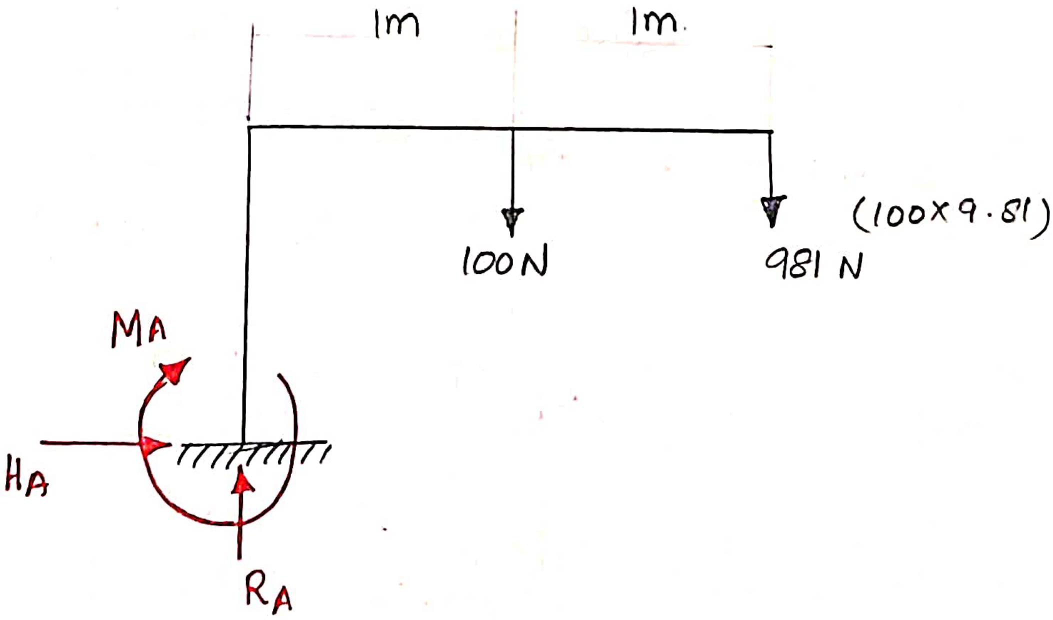
$$\alpha_A = \tan^{-1}\left(\frac{V_A}{H_A}\right)$$

$$= \tan^{-1}\left(\frac{520.87}{161.15}\right) = \boxed{72.8^\circ}$$

Ans.

9) Determine the reactions at the fixed support at A for the loaded frame shown in figure. Take the diameter of pulley as 200 mm.





$\Sigma V = 0$

$$R_A - 100 - 981 = 0$$

$$\boxed{R_A = 1081\ N}$$

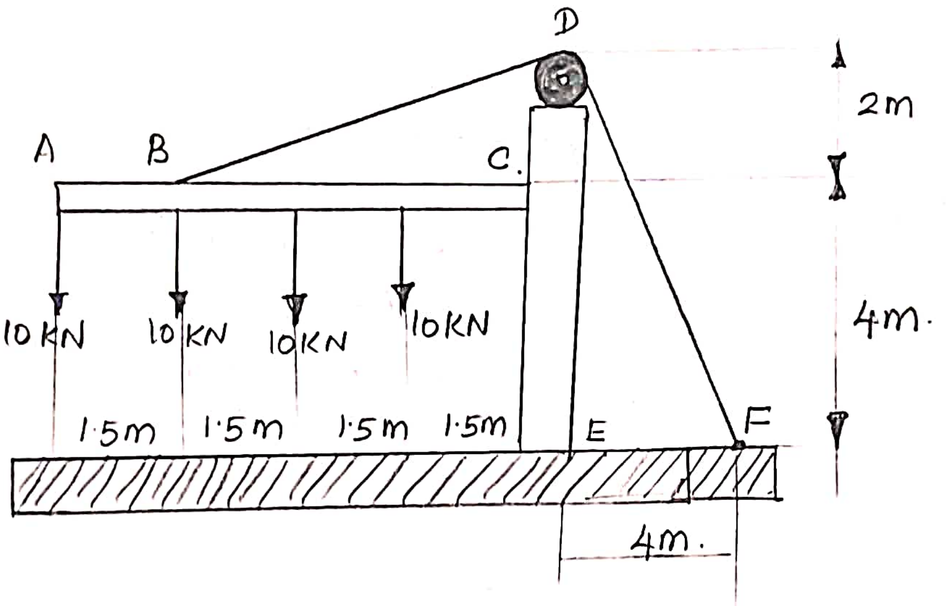
Ans.

$\Sigma H = 0 \Rightarrow \boxed{H_A = 0}$ Ans

$\Sigma M = 0 \Rightarrow$ about 'A' $\quad M_A - (100 \times 1) - (981 \times 2.1) = 0$

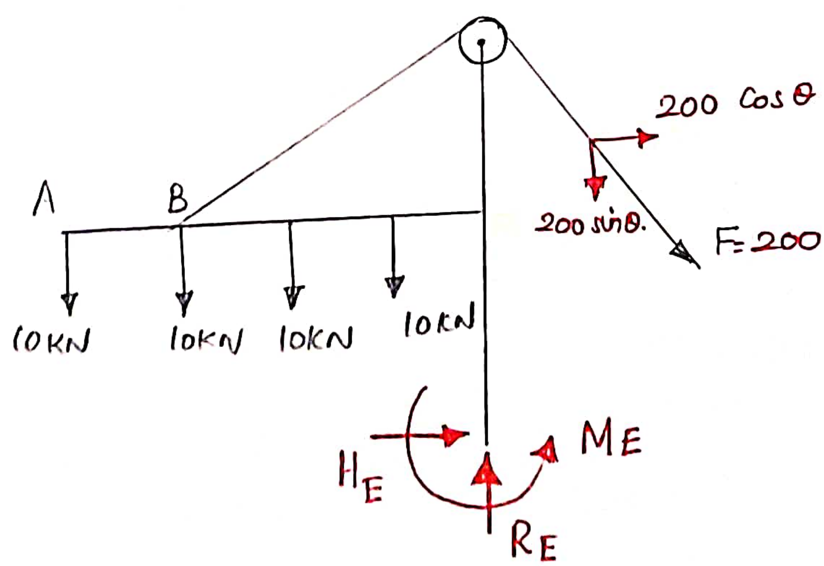
$\boxed{M_A = 2160.1 \text{ N.m}}$. Ans.

10) The frame shown in figure supports a part of the roof of a small building. Knowing that the tension in the cable is 200 KN, determine the reactions at fixed end E.



Solution :-

Tension in the cable is 200 KN.



$\Sigma H = 0 \Rightarrow$ $\qquad$ $DF = \sqrt{4^2+6^2} = 7.2\,m.$

$$H_E + 200 \times \left(\frac{4.0}{7.2}\right) = 0$$

$$\boxed{H_E = -111.11\ KN}$$

$\Sigma V = 0 \Rightarrow \quad R_E - (4 \times 10) - \left[200 \times \left(\frac{6}{7.2}\right)\right] = 0$

$$\boxed{R_E = 206.66\ KN}$$

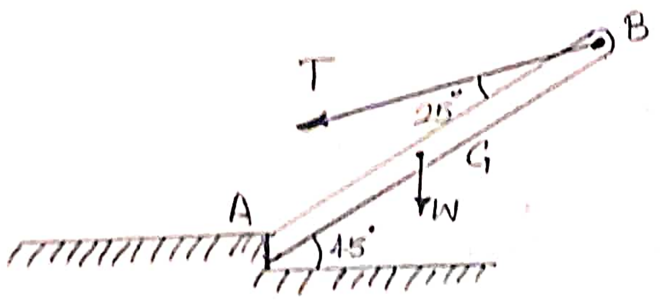
$\Sigma M = 0 \Rightarrow$ about (E)

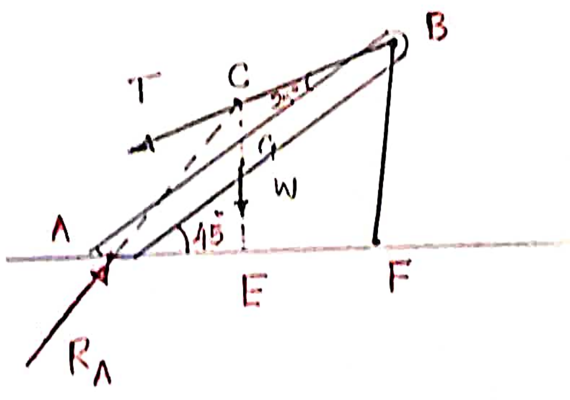
$$-10\times 6 - 10\times 4.5 - 10\times 3.0 - 10\times 1.5 + \frac{6}{7.2}\times 150 \times 4 - M_E = 0$$

$$\boxed{M_E = -350\ KN.m.}$$ (Anticlock wise)

---

11) A man raises a 20 kg. joist of length 4 m, by pulling on a rope. Find the tension 'T' in the rope and the reaction at 'A'.





Solution :-

Mass of the joist (m) = 20 kg

Length of AB = 4 m

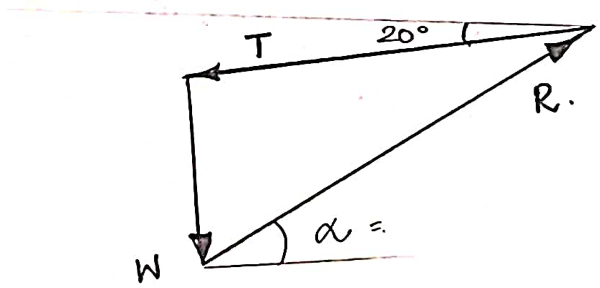
The three forces are :

(i) Weight of joist 20 kg (W)

(ii) force (T) exerted by the rope

(iii) Reaction (R) of the ground at A.

$W = mg = 20 \times 9.81 = 196.2\,N.$



$AF = BF = AB \times \cos 45°$

$= 4 \cos 45$

$= 2.828\ m.$

$EF = AE = \frac{1}{2} AF = 1.414\,m.$

$BD = CD \cos(45° + 25°)$

$= 0.515\,m.$

$CE = DF = BF - BD$

$= 2.828 - 0.515$

$= 2.313\,m$

$\tan \alpha = \frac{CE}{AE}$

$\alpha = \tan^{-1}\left(\frac{2.313}{1.414}\right)$

$\boxed{\alpha = 58.6°}$

Force triangle is drawn & by using the laws of sines

$$\frac{T}{\sin 31.4^\circ} = \frac{R}{\sin 110^\circ} = \frac{W}{\sin 38.6^\circ}$$

$$\frac{196.2}{\sin 38.6^\circ} = \frac{T}{\sin 31.4^\circ}$$

$$T = \frac{196.2}{\sin 38.6} \times \sin 31.4^\circ = \boxed{163.85\,N}$$

Ans.

$$\frac{196.2}{\sin 38.6} = \frac{R}{\sin 110^\circ}$$

$$R = \frac{196.2}{\sin 38.6} \times \sin 110^\circ = \boxed{295.5\ N}$$

Ans.

REACTIONS ON FRAME

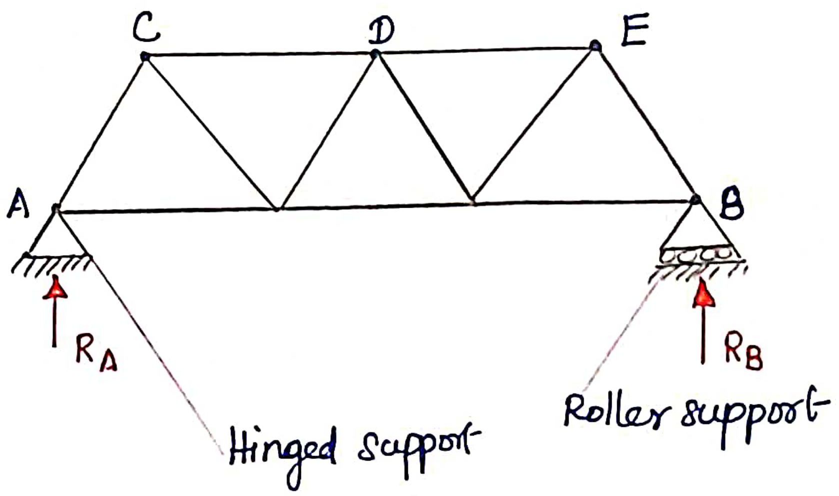
The frames are generally supported

(i) on a roller support (or)

(ii) on a hinged support.

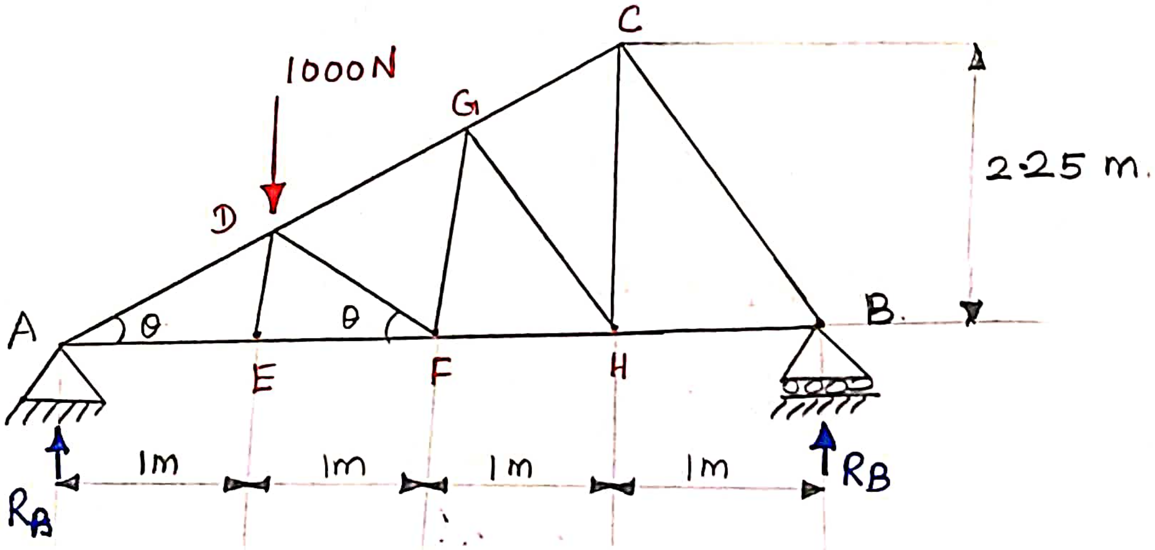
If the frame is roller support → Line of action of reaction will be at right angle to the roller base.

If the frame is hinged support → Line of action will depend upon the load system on the frame.



Problems on Reactions of Frame.

1) A plane truss is loaded and supported as shown in figure. Determine the reactions of the truss.



Solution :-

$$\Sigma M_A = 0 \Rightarrow (R_B \times 4) - (1000 \times 1) = 0$$

$$4 R_B = 1000$$

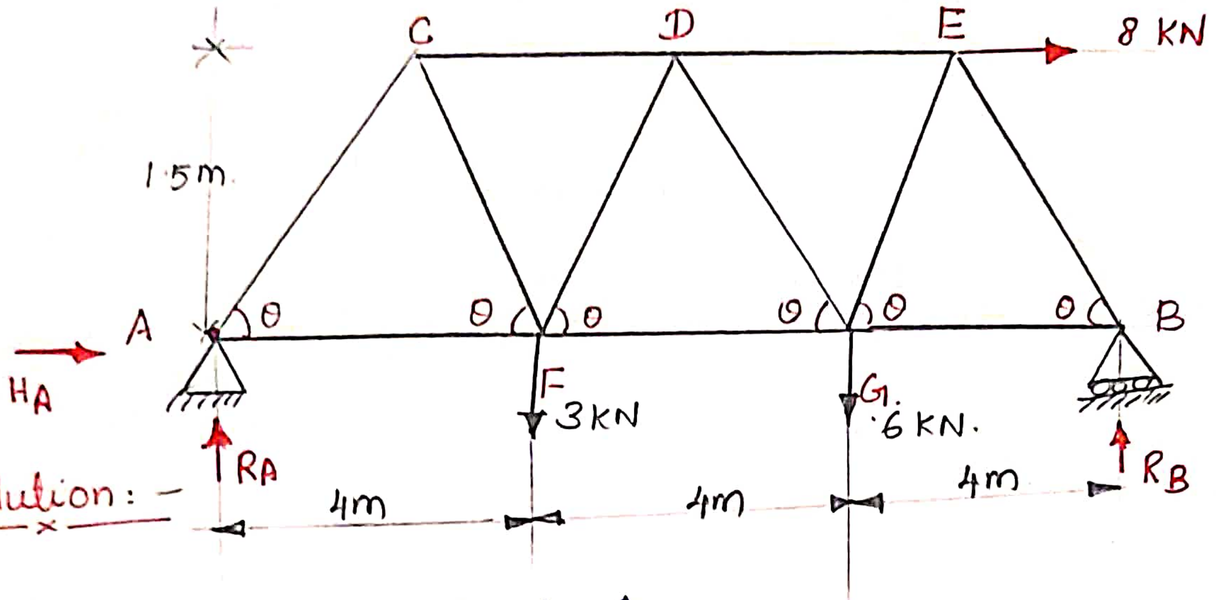
$$\boxed{R_B = 250\ N} \quad \text{--- (1)}$$

$$\Sigma V = 0 \Rightarrow \quad R_A + R_B = 1000$$

from (1)

$$\boxed{R_A = 750\ N}$$

2) Determine the forces in the truss shown in figure which is subjected to horizontal and vertical loads. Mention the nature of forces in each case.



Solution :-

Taking moment about A,

$$R_B \times 12 = (8 \times 1.5) + (3 \times 4) + (6 \times 8) = 72$$

$$12\,R_B = 72$$

$$\boxed{R_B = 6\text{ KN}}$$ Ans

$\Sigma F_x = 0 \Rightarrow \Sigma H = 0$ $H_A + 8 = 0$

$$\boxed{H_A = -8\text{ KN}}$$ Ans

$\Sigma F_y = 0 \Rightarrow \Sigma V = 0 \Rightarrow$ $R_A + R_B - 6 - 3 = 0$

$$R_A = 9 - 6$$

$$\boxed{R_A = 3\text{ KN}}$$ Ans.

TYPES OF EQUILIBRIUM

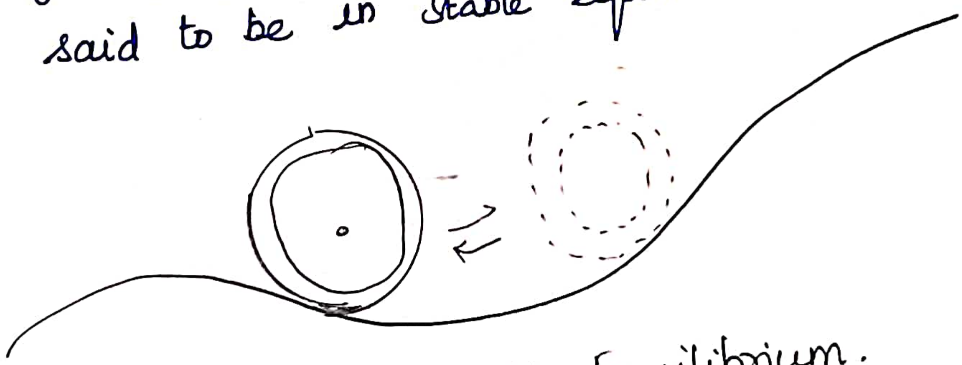
The stability of body is determined by the position of the centre of gravity of that body. Thus centre of gravity of a body controls its equilibrium.

In general we have three types of equilibrium, they are

1. Stable Equilibrium
2. Unstable Equilibrium
3. Neutral Equilibrium

Stable Equilibrium :-

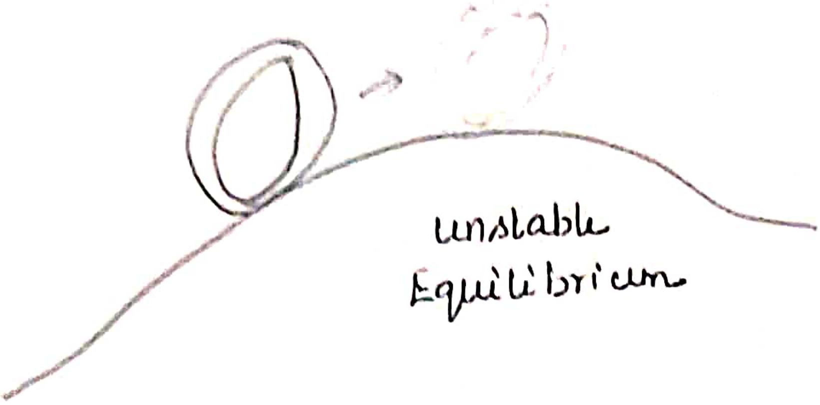
If a body returns back to its original position after being slightly displaced from its rest position, the body is said to be in stable equilibrium.



Stable Equilibrium.

Unstable Equilibrium :-

If a body does not return back to its original position and moves further apart after being slightly displaced, from its rest position, the body is said to be in unstable equilibrium.



unstable Equilibrium

Neutral Equilibrium :-

If a body occupies a position and remains at rest in this position after being slightly displaced from the rest position, the body is said to be in neutral equilibrium.

Neutral Equilibrium.

VECTOR PRODUCT OF TWO VECTORS

The cross or Vector product of two vectors $\vec{P}$ and $\vec{Q}$ is defined as the product of the magnitude of the two vectors and the sine of their included angle. The resultant vector is represented by $\vec{R}$

$$\vec{R} = \vec{P} \times \vec{Q}$$

where $\vec{P}$ and $\vec{Q}$ are the vectors on a same plane, then $\vec{R}$ is normal to the plane of $\vec{P}$ and $\vec{Q}$.

If $\hat{n}$ is the unit vector, which gives the direction of $\vec{R}$ then the cross product can be written as

$$\vec{R} = \vec{P} \times \vec{Q} = (PQ \sin\theta)\,\hat{n}$$

Also the cross vector product is not commutative.

$$\vec{P} \times \vec{Q} \neq \vec{Q} \times \vec{P}$$

$$\vec{P} \times \vec{Q} = -\vec{Q} \times \vec{P}$$

If $\vec{P} = P_x i + P_y j + P_z k$

$\vec{Q} = Q_x i + Q_y j + Q_z k$

$$\boxed{\vec{P} \times \vec{Q} = \begin{vmatrix} i & j & k \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix}}$$

$$\vec{P} \times \vec{Q} = i(P_y Q_z - Q_y P_z) - j(P_x Q_z - Q_x P_z) + k(P_x Q_y - Q_x P_y)$$

The following laws hold good for cross products, where m is a scalar.

$$\vec{P} \times (\vec{Q} + \vec{S}) = \vec{P} \times Q + \vec{P} \times \vec{S}$$

$$m(\vec{P} \times \vec{Q}) = m\vec{P} \times \vec{Q} = \vec{P} \times (m\vec{Q})$$

Note:-

(i) $i \times i = 0$

$j \times j = 0$

$k \times j = 0$

(ii)

$$i \times j = \begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = i(0) - j(0) + k(1) = k$$

similarly

$j \times k = i$

$k \times i = j$

(iii)

$$j \times i = \begin{vmatrix} i & j & k \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix} = i(0) - j(0) + k(-1) = -k$$

Similarly

$k \times j = -i$

$i \times k = -j$

(iv) if $\vec{A} \times \vec{B} = 0$ then $\vec{A}$ and $\vec{B}$ are parallel to each other.

1) Find dot product and cross product of the following Vectors. $\vec{P} = i + 2j - 3k$ and $\vec{Q} = 4i - 5j + 6\vec{k}$.

Solution:-

Dot product $\vec{P} \cdot \vec{Q} = (1 \times 4) + (2 \times (-5)) + [(-3) \times 6]$

$= 4 - 10 - 18 = \boxed{-24.}$ Ans.

Cross product $\vec{P} \times \vec{Q} = \begin{vmatrix} i & j & k \\ 1 & 2 & -3 \\ 4 & -5 & 6 \end{vmatrix}$

$= i(12-15) - j(6+12) + k(-5-8)$

$\boxed{\vec{P} \times \vec{Q} = -3i - 18j - 13k}$ Ans.

---

2) If $\vec{A} = 4\vec{i} + 8\vec{j} - 14\vec{k}$

$\vec{B} = 6\vec{i} + 3\vec{j} - 2\vec{k}$

Find (i) $2\vec{A} + 5\vec{B}$

(ii) $2\vec{A} \cdot 3\vec{B}$

(iii) $3\vec{A} \times 4\vec{B}$

(iv) $(2\vec{A} \times 3\vec{B}) \cdot (\vec{A} \times 2\vec{B})$

Solution:-

(i) $2\vec{A} + 5\vec{B}$

$2\vec{A} = 2(4\vec{i} + 8j - 14k) = 8i + 16j - 28k$

$5\vec{B} = 5(6i - 3j - 2k) = 30i - 15j - 10k$

$2\vec{A} + 5\vec{B} = (8i + 16j - 28k) + (30i - 15j - 10k)$

$\boxed{= 38i + j - 38k}$ Ans

(ii) $\vec{2A} \cdot \vec{3B}$

$\vec{2A} = 2(4i + 8j - 14k) = 8i + 16j - 28k$

$\vec{3B} = 3(6i - 3j - 2k) = 18i - 9j - 6k$

$\vec{2A} \cdot \vec{3B} = (8i + 16j - 28k) \cdot (18i - 9j - 6k)$

$= 144 - 144 + 12$

$\boxed{= 12}$ Ans.

(iii) $\vec{3A} \times \vec{4B}$

$\vec{3A} = 3(4i + 8j - 14\vec{k}) = 12i + 24j - 42k$

$\vec{4B} = 4(6i - 3j - 2\vec{k}) = 24i - 12j - 8k$

$$\vec{3A} \times \vec{4B} = \begin{vmatrix} i & j & k \\ 12 & 24 & -42 \\ 24 & -12 & -8 \end{vmatrix}$$

$= i[(24 \times -8) - (-12 \times -42)] - j[(12 \times -8) - (-42 \times 24)]$

$+ k[(-12 \times 12) - (24 \times 24)]$

$\boxed{= -696i - 912j - 720k}$ Ans.

(iv) $(\vec{2A} \times \vec{3B}) \cdot (\vec{A} \times \vec{2B})$

$\vec{2A} = 8i + 16j - 28k$

$\vec{3B} = 18i - 9j - 6k$

$$\vec{2A} \times \vec{3B} = \begin{vmatrix} i & j & k \\ 8 & 16 & -28 \\ 18 & -9 & -6 \end{vmatrix}$$

$= -348i - 456j - 360k$

$$\vec{A} \times 2\vec{B} = \begin{vmatrix} i & j & k \\ 4 & 8 & -14 \\ 12 & -6 & -4 \end{vmatrix}$$

$$= i\left[(8 \times (-4)) - (-6 \times -14)\right] - j\left[(-4 \times 4) - (12 \times -4)\right]$$
$$+ k\left[(-6 \times 4) - (12 \times 8)\right]$$

$$= -116\,i + 152\,j - 120\,k$$

$$(2\vec{A} \times 3\vec{B}) \cdot (\vec{A} \times 2\vec{B}) = (-348\,i - 456\,j - 360\,k) \cdot (-116\,i + 152\,j - 120\,k)$$

$$= 40368 + 69312 + 43200$$

$$\boxed{= 152880} \text{ Ans.}$$

SCALAR TRIPLE PRODUCT

Scalar triple product of three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ is defined as,

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

where

$$\vec{A} = A_x i + A_y j + A_z k$$
$$\vec{B} = B_x i + B_y j + B_z k$$
$$\vec{C} = C_x i + C_y j + C_z k$$

1) There are 3 concurrent vectors,

$\vec{A} = 5i + 3j + 4K$ m;

$\vec{B} = 2i + 4j + 3K$ m;

$\vec{C} = -i + 5j + 7K$ m

What will be the volume enclosed by these three concurrent vectors?

Solution:

$$\text{Volume} = \vec{A} \cdot (\vec{B} \times \vec{C}) = \begin{vmatrix} A_x & B_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

$$= \begin{vmatrix} 5 & 3 & 4 \\ 2 & 4 & 3 \\ -1 & 5 & 7 \end{vmatrix}$$

$$= 5(28-15) - 3(14+3) + 4(10+4)$$

$$= 65 - 51 + 56 = \boxed{70\, m^3} \text{ Ans.}$$

VECTOR TRIPLE PRODUCT

The vector triple product is a vector quantity and will appear quite often in studies of dynamics, vector triple product of three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ is defined as follows.

$$\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A} \cdot \vec{C})\, \vec{B} - (\vec{A} \cdot \vec{B})\, \vec{C}$$

$$(\vec{A} \times \vec{B}) \times \vec{C} = (\vec{C} \cdot \vec{A})\, \vec{B} - (\vec{C} \cdot \vec{B})\, \vec{A}$$

From the above two results, obviously we come to know that,

$$\vec{A} \times (\vec{B} \times \vec{C}) \neq (\vec{A} \times \vec{B}) \times \vec{C}$$

1) If Vector $\vec{A} = 5i + 3j + 2\vec{K}$

$\vec{B} = i - j - 2K$

a) Find $\vec{A} \times \vec{B}$ and the unit vector along it

(b) Find the included angle between vector $\vec{A}$ the vector resulting from the cross product. (Vector $\vec{A} \times \vec{B}$)

Solution :-

(a) $$\vec{A} \times \vec{B} = \begin{vmatrix} i & j & K \\ 5 & 3 & 2 \\ 1 & -1 & -2 \end{vmatrix}$$

$$= i(-6+2) - j(-10-2) + K(-5-3)$$

$$= -4i + 12j - 8K$$

Magnitude of $\vec{A} \times \vec{B} = \sqrt{16+144+64} = 14.96$

unit vector along $\vec{A} \times \vec{B} = \frac{-4}{14.96} i + \frac{12}{14.96} j - \frac{8}{14.96} K$

$$\boxed{= -0.267i + 0.802j - 0.535K}$$

Ans.

(b) Included angle between vectors $\vec{A}$ and $\vec{A} \times \vec{B}$

$$\vec{A} \times (\vec{A} \times \vec{B}) = \begin{vmatrix} i & j & K \\ 5 & 3 & 2 \\ -4 & 12 & -8 \end{vmatrix}$$

$$= i(-24-24) - j(-40+8) + K(60+12)$$

$$\vec{A} \times (\vec{A} \times \vec{B}) = -48i + 32j + 72K$$

Magnitude of $\vec{A} \times (\vec{A} \times \vec{B}) = \sqrt{(-48)^2 + (32)^2 + (72)^2}$

$$= 92.26$$

$$\sin\theta = \frac{|\vec{A} \times (\vec{A} \times \vec{B})|}{|\vec{A}||\vec{A} \times \vec{B}|}$$

$$= \frac{92.26}{(6.16)(14.96)}$$

$$\sin\theta = 1.0$$

$\boxed{\theta = 90^\circ}$ Ans.

Moment of a Force About a Point in three Dimensions.

$$\vec{M_o} = \vec{r} \times \vec{F} \quad \text{——} ⓐ$$

$$= (x_A \vec{i} + y_A \vec{j} + z_A \vec{k}) \times (F_x \vec{i} + F_y \vec{j} + F_z \vec{k}) \quad \text{——} b$$

where $\vec{F}$ is the force, $\vec{r}$ is the position vector:

Let Moment Vector $\vec{M_o} = M_x \vec{i} + M_y \vec{j} + M_z \vec{k}$ —— c

$$\vec{M_o} = \vec{r} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_A & y_A & z_A \\ F_x & F_y & F_z \end{vmatrix}$$

$$\vec{M_o} = (F_z y_A - F_y z_A)i + (F_x z_A - F_z x_A)j + (F_y x_A - F_x y_A)k \quad \text{——} ⓓ$$

Equaling eqn (c) and (d)

$$M_x = F_z y_A - F_y z_A$$

$$M_y = F_x z_A - F_z x_A$$

$$M_z = F_y x_A - F_x y_A$$

Where M_x, M_y and M_z are Scalar quantity of $\vec{M}$ about x, y and z axes respectively.

Magnitude of moment

Moment Vector $\vec{M} = M_x i + M_y j + M_z k$

$$M = \sqrt{M_x^2 + M_y^2 + M_z^2}$$

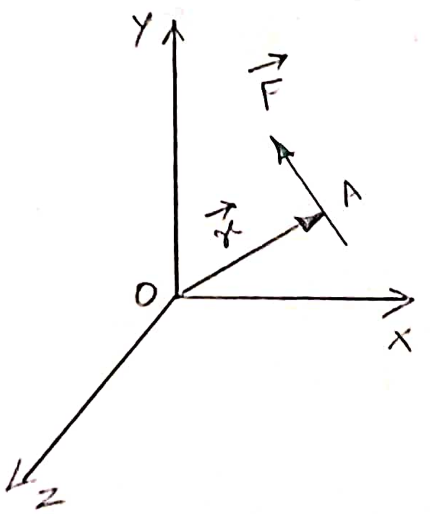
Direction of Moment $\vec{M}$

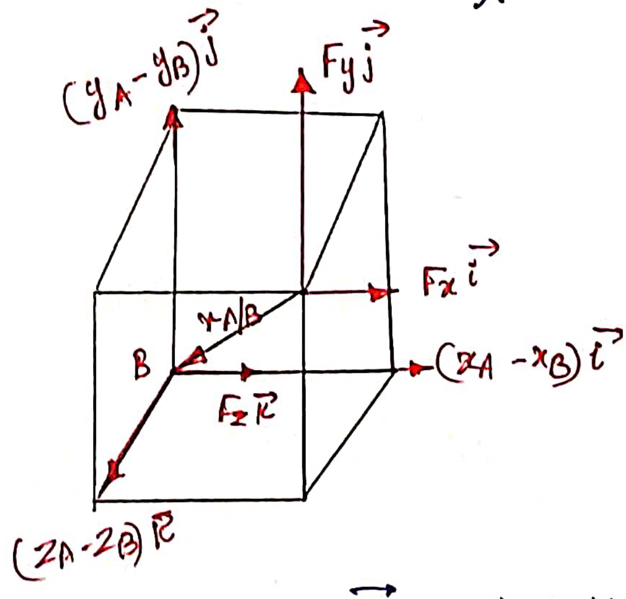
Let the moment vector $\vec{M}$ makes angles of ϕ_x, ϕ_y and ϕ_z about x, y and z axes respectively.

$$\cos \phi_x = \frac{M_x}{M} \quad ; \quad \phi_x = \cos^{-1}\left(\frac{M_x}{M}\right)$$

$$\cos \phi_y = \frac{M_y}{M_y} \quad ; \quad \phi_y = \cos^{-1}\left(\frac{M_y}{M}\right)$$

$$\cos \phi_z = \frac{M_z}{M} \quad ; \quad \phi_z = \cos^{-1}\left(\frac{M_z}{M}\right)$$





Similarly moment of the forces $\vec{F}$ passing through $A(x_A, y_A, z_A)$ about another point $B(x_B, y_B, z_B)$ can be expressed as,

$$\vec{M}_{BA} = \vec{r}_{A/B} \times \vec{F}$$

$$\vec{r}_{A/B} = \vec{r}_{OA} - \vec{r}_{OB} = (x_A - x_B)\,i + (y_A - y_B)\,j + (z_A - z_B)\,k$$

$$\boxed{\vec{M}_B = \begin{vmatrix} i & j & \vec{k} \\ (x_A - x_B) & (y_A - y_B) & (z_A - z_B) \\ F_x & F_y & F_z \end{vmatrix}}$$

---

1) A force $\vec{F} = 6i + 2j - 4k$ passes through a point A, whose position vector is $2i - j + 4.5k$. Find the moment of the force about point 'B' whose position vector is $3i - 1.5j - 3.5k$.

Solution :-

$$\vec{F} = 6i + 2j - 4k$$

Position vector of $\vec{A} = 2i - j + 4.5k$

Position vector of $\vec{B} = 3i - 1.5j - 3.5k$

$$\vec{M} = \vec{r}_{A/B} \times \vec{F}$$

$$\vec{r}_{A/B} = \vec{OA} - \vec{OB} = (2i - j + 4.5k) - (3i - 1.5j - 3.5k)$$

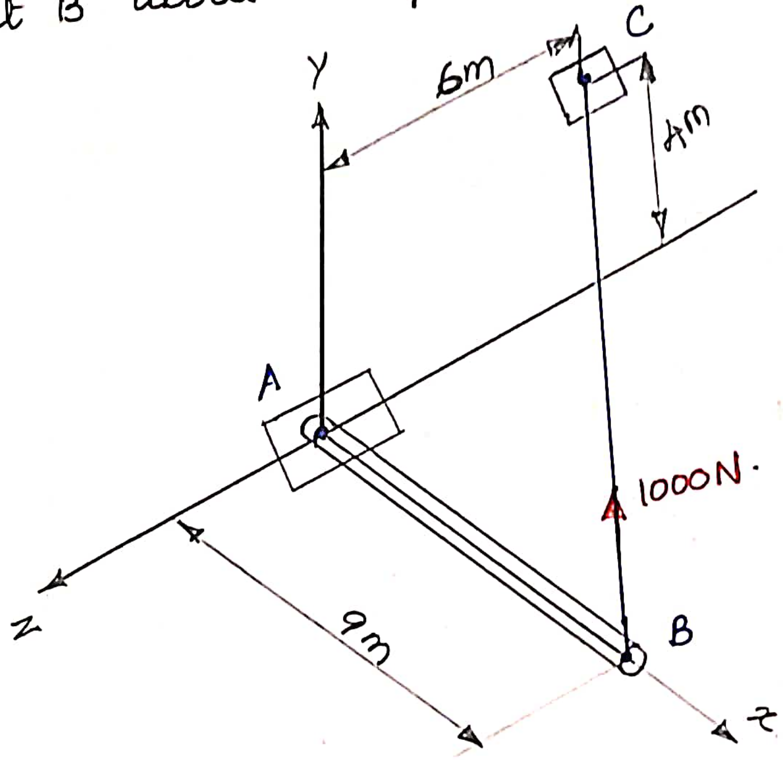
$$= -i + 0.5j + 8k$$

$$\vec{M} = \vec{r}_{A/B} \times \vec{F} = \begin{vmatrix} i & j & k \\ -1 & 0.5 & 8 \\ 6 & 2 & -4 \end{vmatrix}$$

$$= i(-2 - 16) - j(4 - 48) + k(-2 - 3)$$

$$\boxed{\vec{M} = -18i + 44j - 5k}$$ Ans.

2) 9m rod AB shown in figure has a fixed end at A. A steel cable is stretched from B to point C on the vertical wall. If the tension in the cable BC is 1000 N, find the moment of A of the force exerted by the cable at B about the point 'A'.



Solution:-

Magnitude of F = 1000 N

Co-ordinates of A (0, 0, 0) ; B (9, 0, 0) ; C (0, 4, -6)

$$M_A = \vec{r}_{B/A} \times \vec{F}$$

$$\vec{r}_{B/A} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$x = x_B - x_A = 9$$

$$y = y_B - y_A = 0$$

$$z = z_B - z_A = 0$$

$$\vec{r}_{BA} = \vec{9i}$$

Force Vector $\vec{F} = F\hat{n}$

$$\hat{n} = \frac{\vec{BC}}{BC}$$

$$\vec{BC} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$x = x_C - x_B = -9$$

$$y = y_C - y_B = 4$$

$$z = z_C - z_B = -6$$

$$\vec{BC} = -9\vec{i} + 4\vec{j} - 6\vec{k}$$

$$BC = \sqrt{(-9)^2 + (4)^2 + (-6)^2} = 11.53$$

$$\hat{n} = \frac{1}{11.53}\left(-9\vec{i} + 4\vec{j} - 6\vec{k}\right)$$

$$\vec{F} = \frac{1000}{11.53}\left(-9\vec{i} + 4\vec{j} - 6\vec{k}\right)$$

$$= -780.57\vec{i} + 346.92\vec{j} - 520.38\vec{k}$$

$$M_A = \vec{r}_{A/B} \times \vec{F}$$

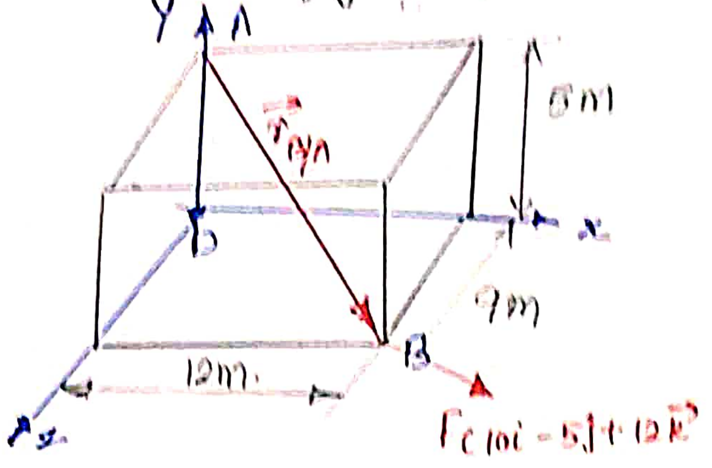
$$= \begin{vmatrix} i & j & \vec{k} \\ 9 & 0 & 0 \\ -780.57 & 346.92 & -520.38 \end{vmatrix}$$

$$= -j(-520.38 \times 9) + \vec{k}(9 \times 346.92)$$

$$\boxed{= 4683.42\vec{j} + 3122.28\vec{k} \quad \text{Nm}}$$

Ans.

2) Find the moment of the force $\vec{F}$ acting at B about point 'A' in the figure given below.



Solution :-

Co-ordinates of A $(0, 5, 0)$; B $(12, 0, 9)$

Force $\vec{F} = 10\vec{i} - 5\vec{j} + 12\vec{k}$

$M_A = \vec{r}_{B/A} \times \vec{F}$

$\vec{r}_{B/A} = x\vec{i} + y\vec{j} + z\vec{k}$

$x = x_B - x_A = 12 - 0 = +12$

$y = y_B - y_A = 0 - 5 = -5$

$z = z_B - z_A = 9 - 0 = 9.$

$\vec{r}_{B/A} = 12\vec{i} - 5\vec{j} + 9\vec{k}$

$M_A = (12\vec{i} - 5\vec{j} + 9\vec{k}) \times (10\vec{i} - 5\vec{j} + 12\vec{k})$

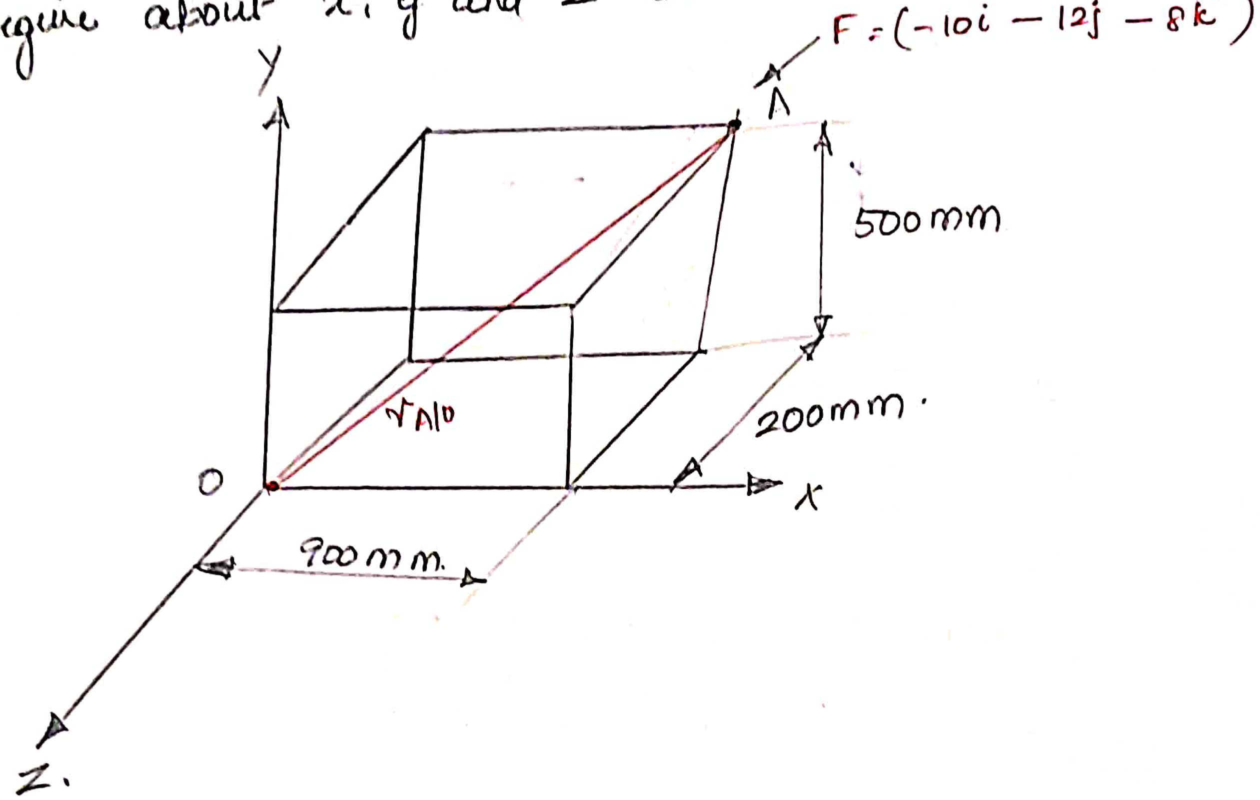
$$= \begin{vmatrix} i & j & k \\ 12 & -5 & 9 \\ 10 & -5 & 12 \end{vmatrix}$$

$M_A = (-60 + 45)\vec{i} - (144 - 90)\vec{j} + (-60 + 50)\vec{k}$

$\boxed{M_A = -15\vec{i} - 54\vec{j} - 10\vec{k} \text{ N.m}}$ Ans.

Moment of the Force about a given Axes.

1) Determine the moments of the forces $\vec{F}$ shown in figure about x, y and z axis.



Solution: -

$$\vec{F} = -10\vec{i} - 12\vec{j} - 8\vec{k}$$

Co-ordinates of A = (900, 500, −200)

B = (900, 500, 0)

First moment of force about O should be determined.

$$\vec{M_O} = \vec{r_{AO}} \times \vec{F}$$

$$\vec{r_{A/O}} = x\vec{i} + y\vec{j} + z\vec{k}$$

$x = 0.9 - 0 = 0.9$

$y = 0.5 - 0 = 0.5$

$z = -0.2 - 0 = -0.2$

$$\vec{r_{OA}} = 0.9\vec{i} + 0.5\vec{j} - 0.2\vec{k}$$

$$\vec{F} = -10\vec{i} - 12\vec{j} - 8\vec{k}$$

$$\vec{M_O} = (0.9\vec{i} + 0.5\vec{j} - 0.2\vec{k}) \times (-10\vec{i} - 12\vec{j} + 8\vec{k})$$

$$\begin{vmatrix} i & j & k \\ 0.9 & 0.5 & -0.2 \\ -10 & -12 & -8 \end{vmatrix}$$

$$\vec{M_O} = (4 - 2.4)\vec{i} - (7.2 - 2)\,j + (-10.8 + 5)$$

$$\vec{M_O} = 1.6\vec{i} - 5.2\vec{j} - 5.8\vec{k}$$

Moment of force $\vec{F}$ about OX axis = $\boxed{1.6 \text{ Nm}}$ clockwise

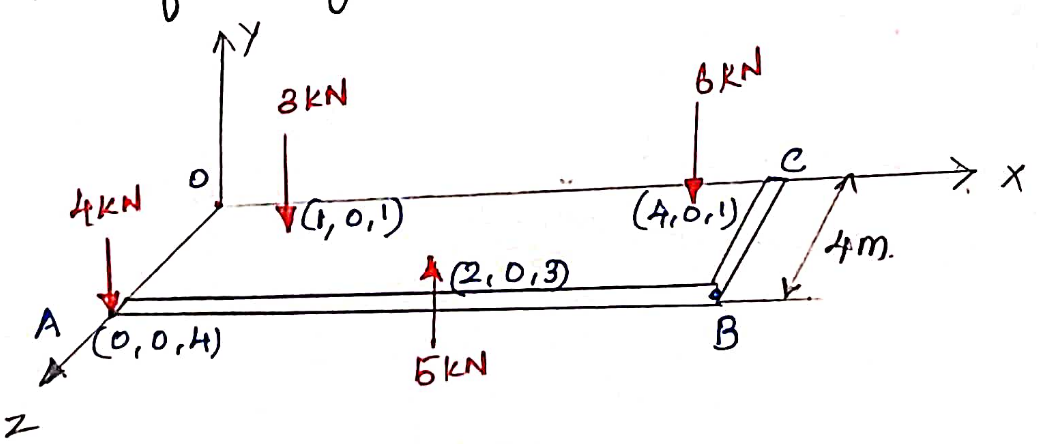
" " " OY axis = $\boxed{-5.2 \text{ Nm}}$ (ACW)

" " " OZ axis = $\boxed{-5.8 \text{ Nm}}$ (ACW)

Ans.

Reactions at supports and connections for a three dimensional structure.

1) A 4m x 5m slab carries four forces normal to it as shown in figure. Determine (i) the equivalant action which can be applied only at point O (ii) the single resultant of the force system.



Solution :-

The resultant force of the system is $= -3\hat{j} - 4\hat{j} - 6\hat{j} + 5\hat{j}$
$= -8\hat{j}$

ie a force of magnitude 8 kN acting downward is the resultant.

(i) To find the equivalant action which can be applied at 'O'

The resultant 8 kN downwards is to be applied at 'O' with a moment. The moment must be equal to the moment exerted by the forces in the present system.

$$\text{Moment} = -3\hat{j} \times (\hat{i} + \vec{k}) - 6\hat{j} \times (4\vec{i} + \vec{k}) - 4\hat{j}(4\vec{i}) + 5\hat{j}(2\hat{j} + 3\vec{k})$$

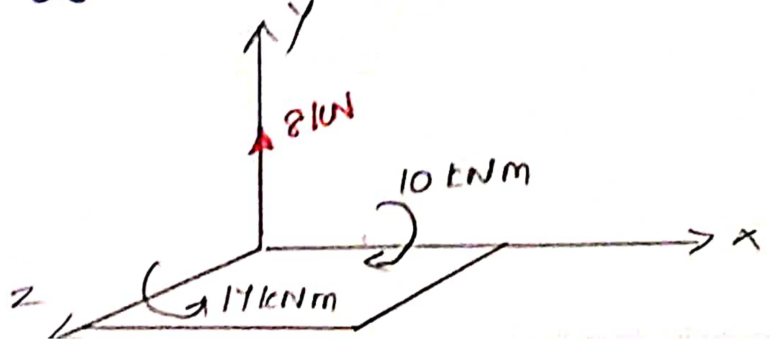
$$= \begin{vmatrix} i & j & \vec{k} \\ 0 & -3 & 0 \\ 1 & 0 & 1 \end{vmatrix} - \begin{vmatrix} i & j & k \\ 0 & -6 & 0 \\ 4 & 0 & 1 \end{vmatrix} + \begin{vmatrix} i & j & \vec{k} \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 0 & 5 & 0 \\ 2 & 0 & 3 \end{vmatrix}$$

$$\vec{M} = -3(\vec{i} - \vec{k}) - 6(\vec{i} - 4\vec{k}) - 4(4\vec{i}) + 5(3\vec{i} - 2\vec{k})$$

$$\vec{M} = -3\vec{i} + 3\vec{k} - 6\vec{i} + 24\vec{k} - 16\vec{i} + 15\vec{i} - 10\vec{k}$$

$$\boxed{\vec{M} = -10\vec{i} + 17\vec{k}}$$

which means, the moment has x component 10 kNm is acw and z component 17 kNm is cw.



(ii) The single resultant force :-

A single force acting in the plane must exert the moment 10 KNm anticlockwise in x plane and 17 KNm clockwise in Z plane.

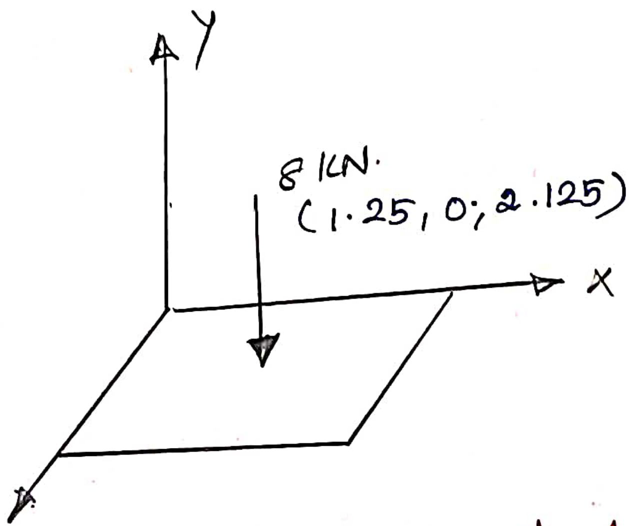
Moment = Force x distance.

The distance resultant 8 kN from OZ axis (x coordinate)

$$O = \frac{\text{Moment}}{\text{force}} = \frac{-10}{-8} = 1.25\text{m}.$$

The Z co-ordinate of the resultant 8kN from O

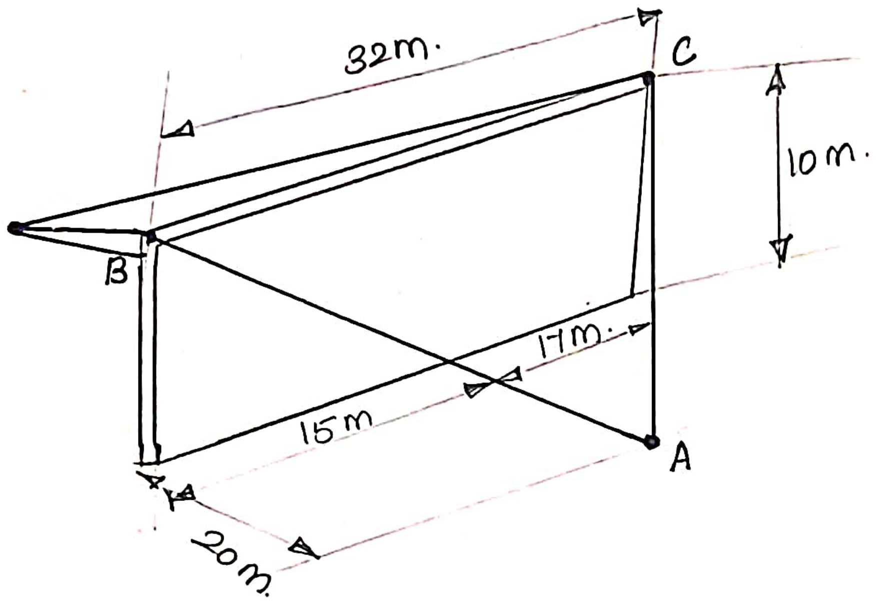
$$= \frac{17}{8} = 2.125 \text{ m}.$$



Single resultant force.

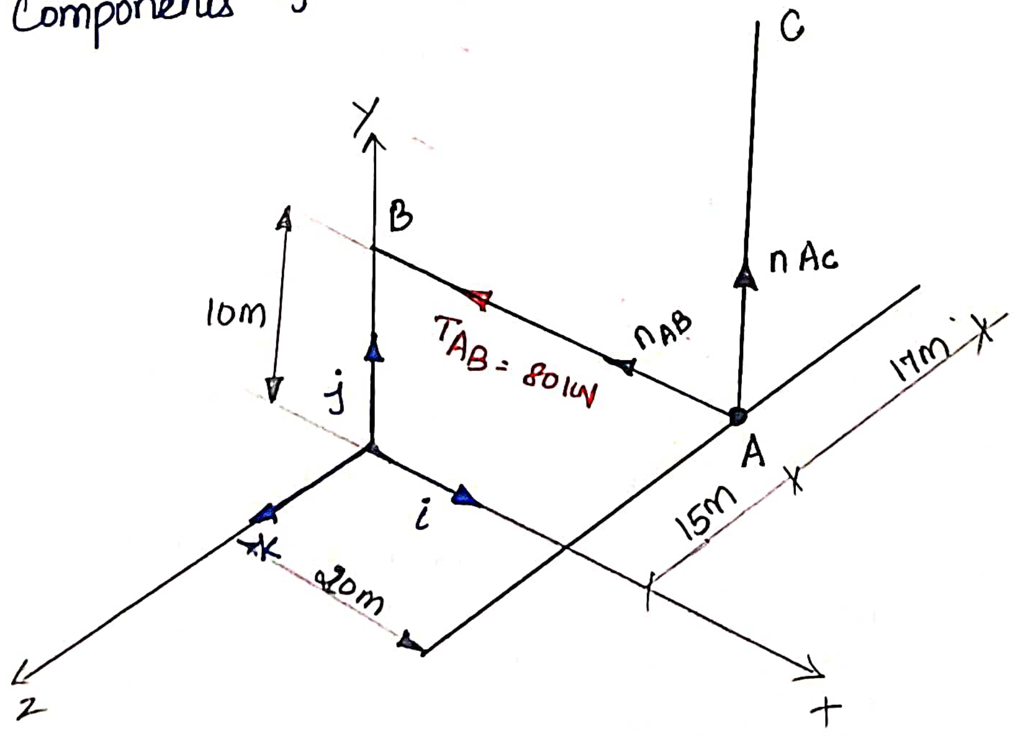
The force of 8 KN acting at (1.25, 0, 2.125) is the single resultant force of the given system.

2) An advertisement boarding is held by two cables shown in figure. If the tension is 8 KN in cable AB and 10 KN in cable AC. Find the magnitude and direction of resultant of the forces caused by both cables on the stake A.



Solution :-

Components of the forces



$\vec{AB} = -20i + 10j + 15k$

$AB = 26.92$ m

$\vec{AC} = -20i + 10j - 17k$

$AC = 28.08$ m

Denoting $\hat{n}_{AB}$ the unit vector along AB

$$\vec{T_{AB}} = T_{AB} \cdot \hat{n}_{AB}$$

$$\vec{T_{AB}} = T_{A.B} \times \frac{\vec{AB}}{AB}$$

$$= 8 \times \frac{(-20i + 10j + 15k)}{26.92}$$

$$\vec{T_{AB}} = (-5.94)i + (2.97)j + 4.46k$$

Similarly $\vec{T_{AC}} = T_{AC} \cdot \hat{n}_{AC}$

$$= T_{AC} \cdot \frac{\vec{AC}}{AC} = 10 \text{ kN} \times \frac{(\vec{AC})}{28.08}$$

$$T_{AC} = (-7.12)i + (3.56)j - (6.05)k$$

Resultant of the forces,

$$\vec{R} = \vec{T_{AB}} + \vec{T_{AC}}$$

$$= -(13.06)i + (6.53)j - (1.59)k$$

Magnitude and direction of the resultant is now determined.

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

$$= \sqrt{(13.06)^2 + (6.53)^2 + (1.59)^2}$$

$$\boxed{R = 14.6\,N}$$ Ans.

$$\cos\theta_x = \frac{R_x}{R} = \frac{-13.06}{14.6} \Rightarrow \boxed{\theta_x = 153.4^\circ}$$

$$\cos\theta_y = \frac{R_y}{R} = \frac{6.53}{14.6} \Rightarrow \boxed{\theta_y = 63.43^\circ}$$

$$\cos\theta_z = \frac{R_z}{R} = \frac{1.59}{14.6} \Rightarrow \boxed{\theta_z = 96.25^\circ}$$ Ans.

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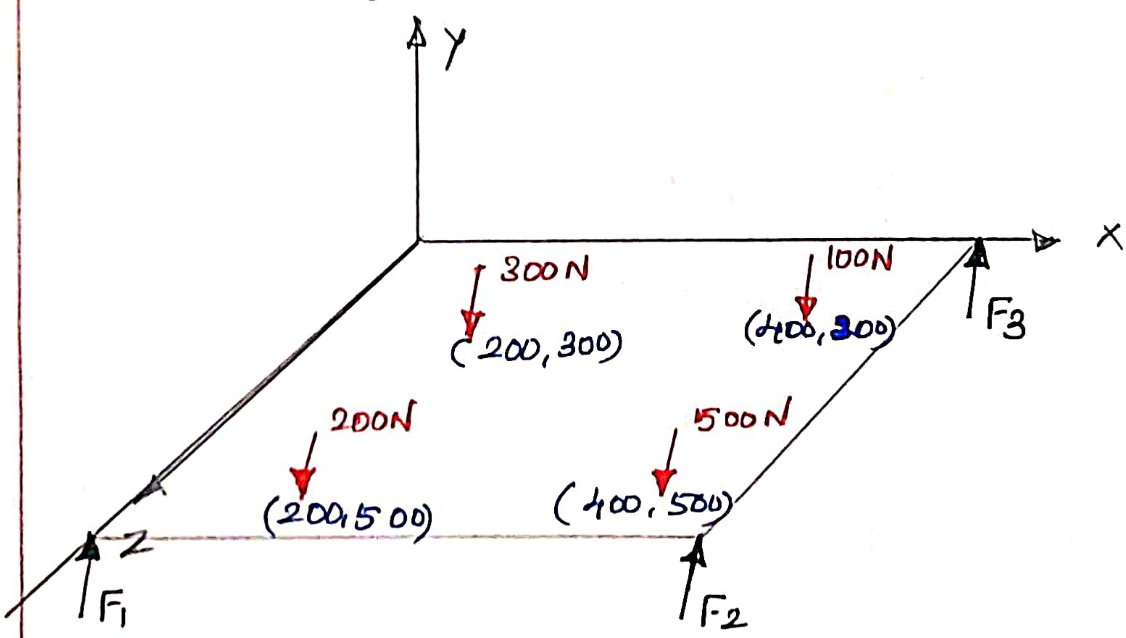
3) A body of 800 mm by 800 mm is kept in equilibrium by forces F_1, F_2, F_3. Four loads are applied as shown in figure. Determine the three forces F_1, F_2 & F_3.



Solution :-

Applying the three equilibrium equations.

$$\Sigma F_y = 0 \Rightarrow F_1 + F_2 + F_3 - 200 - 300 - 100 - 500 = 0$$

$$F_1 + F_2 + F_3 = 1100 \text{ N} \quad \text{——— (1).}$$

Moment about OX axis,

$$\Sigma M_x = 0 \Rightarrow (-F_1 \times 800) - F_2 \times 800 + 200 \times 500 + 300 \times 300$$

$$+ 500 \times 500 + 100 \times 200 = 0$$

$$(F_1 + F_2)\, 800 = 460000$$

$$F_1 + F_2 = 575 \quad \text{——— (2).}$$

Moment about OZ axis,

$$\Sigma M_z = 0 \Rightarrow -F_2 \times 800 + 200 \times 200 + 500 \times 400 + 100 \times 400$$

$$+ 300 \times 200 = 0$$

$$(F_2 + F_3)\, 800 = 340000$$

$$F_2 + F_3 = 425 \quad \text{——— (3)}$$

Solving equation (1), (2) & (3)

We get $F_3 = 1100 - 575 = \boxed{525 \text{ N}}$

$F_2 = 425 - 525 = \boxed{-100 \text{ N}}$

$F_3 = 575 + 100 = \boxed{675 \text{ N}}$

Answers.

UNIT – III

PROPERTIES OF SURFACES AND SOLIDS

CENTRE OF GRAVITY

The centre of gravity of a body is defined as a point through which the entire weight of body acts, irrespective of the orientation of body. It is denoted by C.G. (or) G.

It may be noted that every body has one and only one centre of gravity. It is a well known fact that, all material bodies are attracted by the earth.

The attraction of earth n material bodies is called gravity. Due to this attraction, the earth applies force on all bodies and this force is called gravitational force. This gravitational force is also known as weight of the body. This gravitational force is proportional to the mass of body and it always act vertically downwards.

Since the body is a collection of small particles, such force of gravty acts on each particles are all directed towards the centre of earth. Since we are dealing with the bodies which are very small as compared to the earth, these forces can be assumed to be parallel.

NOTE:-

The Centre of Gravity of solid is given as

$$\bar{x} = \frac{mx}{m} = \frac{m_1x_1 \quad m_2x_2 \quad m_3x_3 \quad ....}{m_1 \quad m_2 \quad m_3 \quad ....}$$

$$\bar{y} = \frac{my}{m} = \frac{m_1y_1 \quad m_2y_2 \quad m_3y_3 \quad ....}{m_1 \quad m_2 \quad m_3 \quad ....}$$

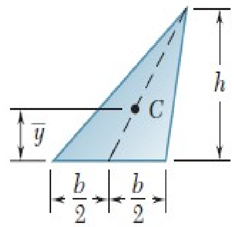
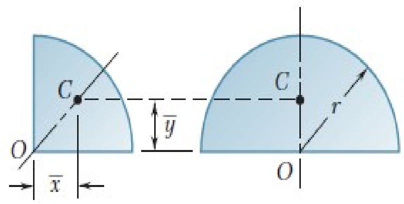
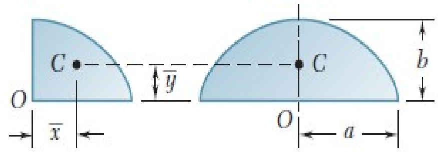
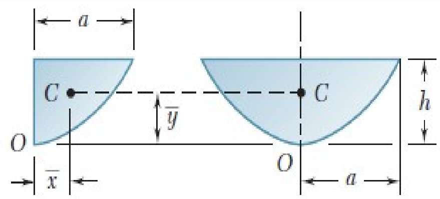
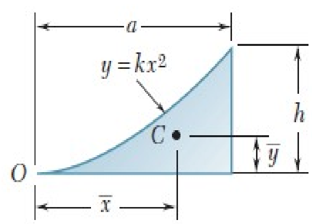
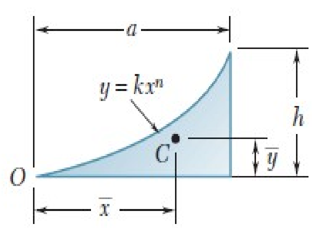
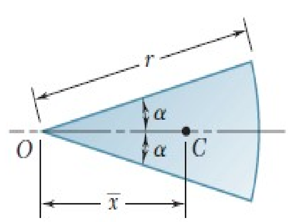
where,

m_1, m_2, m_3 are masses of small elemental strips

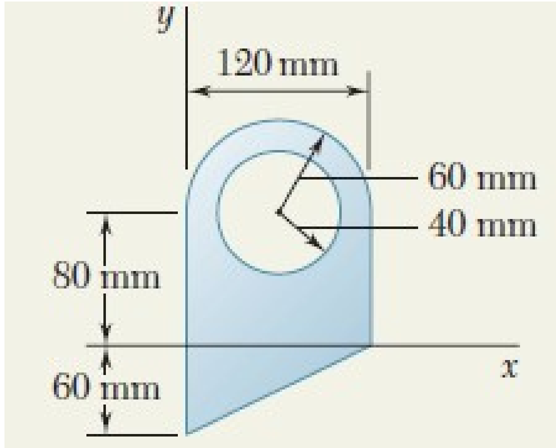
x_1 x_2 x_3 are the respective co-ordinates of masses m_1,m_2, m_3...... on x-axis w.r.t same axis of reference.

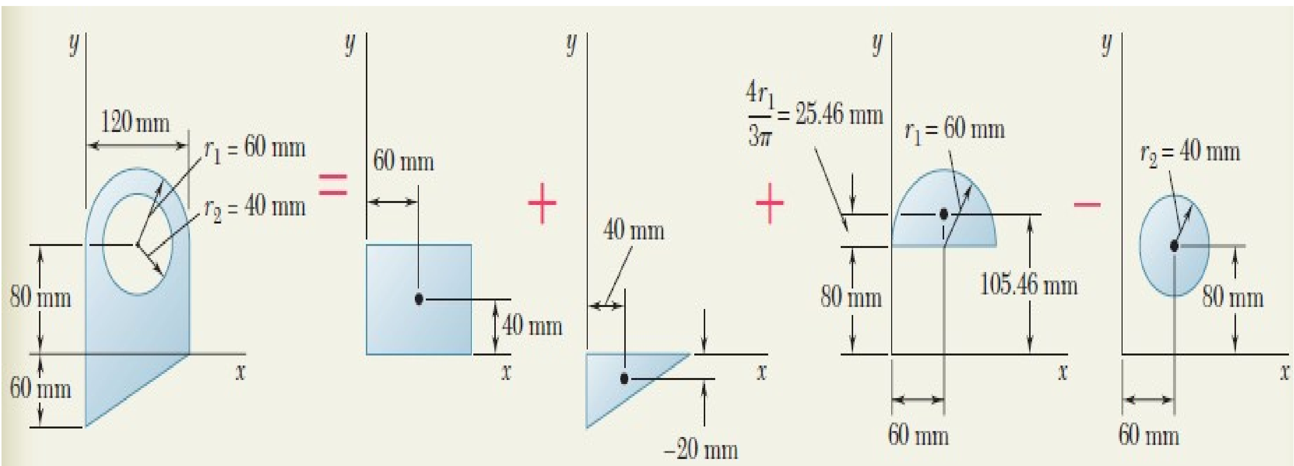
y_1,y_2,y_3, are respective co-ordinate of masses m_1,m_2,m_3 on y-axis w.r.t same axis of reference.

CENTROID BY GEOMETRICAL CONSIDERATIONS

Shape		$\bar{x}$	$\bar{y}$	Area
Triangular area			$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter-elliptical area		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Semielliptical area		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Parabolic spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
General spandrel		$\frac{n+1}{n+2}a$	$\frac{n+1}{4n+2}h$	$\frac{ah}{n+1}$
Circular sector		$\frac{2r\sin\alpha}{3\alpha}$	0	αr^2

For the plane area shown, determine (*a*) the first moments with respect to the x and y axes, (*b*) the location of the centroid.





Component	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
Rectangle	$(120)(80) = 9.6 \times 10^3$	60	40	$+576 \times 10^3$	$+384 \times 10^3$
Triangle	$\frac{1}{2}(120)(60) = 3.6 \times 10^3$	40	-20	$+144 \times 10^3$	-72×10^3
Semicircle	$\frac{1}{2}\pi(60)^2 = 5.655 \times 10^3$	60	105.46	$+339.3 \times 10^3$	$+596.4 \times 10^3$
Circle	$-\pi(40)^2 = -5.027 \times 10^3$	60	80	-301.6×10^3	-402.2×10^3
	$\Sigma A = 13.828 \times 10^3$			$\Sigma \bar{x}A = +757.7 \times 10^3$	$\Sigma \bar{y}A = +506.2 \times 10^3$

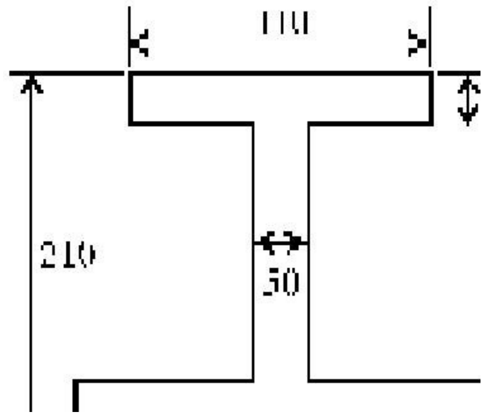
$$Q_x = \Sigma \bar{y}A = 506.2 \times 10^3 \text{ mm}^3 \qquad Q_x = 506 \times 10^3 \text{ mm}^3$$
$$Q_y = \Sigma \bar{x}A = 757.7 \times 10^3 \text{ mm}^3 \qquad Q_y = 758 \times 10^3 \text{ mm}^3$$

b. Location of Centroid. Substituting the values given in the table into the equations defining the centroid of a composite area, we obtain

$$\bar{X}\Sigma A = \Sigma \bar{x}A: \qquad \bar{X}(13.828 \times 10^3 \text{ mm}^2) = 757.7 \times 10^3 \text{ mm}^3$$
$$\bar{X} = 54.8 \text{ mm}$$

$$\bar{Y}\Sigma A = \Sigma \bar{y}A: \qquad \bar{Y}(13.828 \times 10^3 \text{ mm}^2) = 506.2 \times 10^3 \text{ mm}^3$$
$$\bar{Y} = 36.6 \text{ mm}$$

Find the centroid of the I -section shown in fig



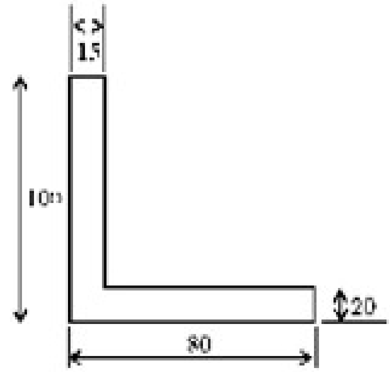
S.No.	Component	Area 'a' (mm^2)	Centroidal distance from 1-1 axis 'y' (mm)	ay (mm^3)
1.	Rectangle ABCD	150 x 60 = 9000	$\frac{60}{2} + 30$	270×10^3
2.	Rectangle EFGH	100 x 50 = 5000	$\frac{100}{2} + 60 = 110$	550×10^3
3.	Rectangle JKLM	110 x 50 = 5500	$\frac{50}{2} + 100 + 60 = 185$	1017.5×10^3
		a = 19500		$ay = 1837.5 \times 10^3$

Distance of centroid from 1-1 axis, $\bar{y} = \frac{ay}{a}$

$$= \frac{1837.5\ 10^3}{19500}$$

y 94.23mm **(Ans)**

Find the center of a gravity of L-Section shown in fig



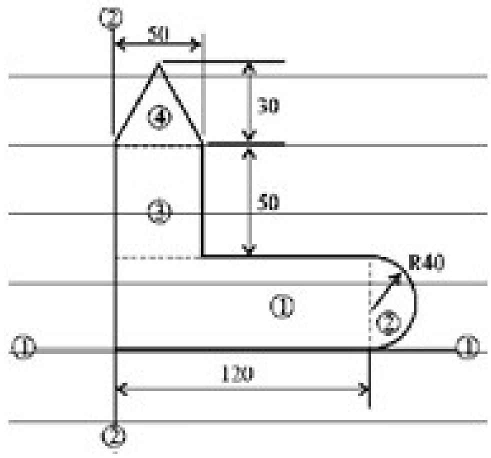
S. No.	Component	Area 'a' (mm^2)	Centroidal distance from 1-1 axis (y)	Centroidal distance from 2-2 axis (x)	ay (mm^3)	ax (mm^3)
1.	Rectangle ABCD	80 x 20 = 1600	$\frac{20}{2} = 10$	$\frac{80}{2} = 10$	16×10^3	64×10^3
2.	Rectangle DEFG	80 x 15 = 1200	$\frac{80}{2} + 20 = 60$	$\frac{15}{2} = 7.5$	72×10^3	9×10^3
		a = 2800			ay $=88 \times 10^3$	ax $=73 \times 10^3$

Distance of centroid from reference axis 2-2, $\bar{x} = \frac{ax}{a}$

$$= \frac{73\ 10^3}{2800}$$

$\bar{x}$ **26.07 mm** (Ans)

Find the center of a gravity area shown in fig



S. No.	Component	Area 'a' (mm^2)	Centroidal distance from 1-1 axis (y)	Centroidal distance from 2-2 axis (x)	ax (mm^3)	ay (mm^3)
1.	Rectangle	6x1=6	$\frac{6}{2} = 3$	$\frac{1}{2} = 0.5$	18	3
2.	Rectangle	9x1.5=13.5	$\frac{1.5}{2} = 0.75$	$\frac{9}{2} + 1 = 5.5$	10.125	74.25
3.	Triangle	$\frac{1}{2} = bxh$ $= \frac{1}{2}\, 2.5 x 6$ = 7.5	$\frac{b}{3} + 1.5$ $= \frac{2.5}{3} + 1.5$ =2.33	$\frac{b}{3} + 1$ $= \frac{6}{3} + 1$ = 3	17.475	22.5
		a = 27			ax = 45.6	ay = 99.75

Centoridal distance from 2-2 axis,

$$\bar{x} = \frac{ax}{a}$$

$$= \frac{45.6}{27}$$

$\bar{x}$ **= 1.69cm** **(Ans)**

Centroidal distance from 1-1 axis,

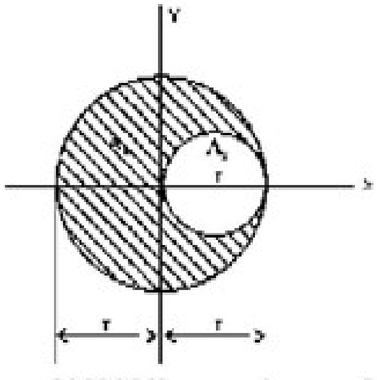
$$\bar{y} = \frac{ay}{a}$$

$$= \frac{99.75}{27}$$

$\bar{y}$ **= 3.69 cm** **(Ans)**

Find the Centroid of area excluding a circle of radius 2 is removed from the circle of radius ' r ' as shown in

Solution :



The given section is symmetrical about x-x axis Hence the Centroid lies on x-axis Area of larger circle, $A_1 = r^2 r^2$

Area of smaller circle, $A_2 = \frac{r_2}{2} \; \frac{r^2}{4}$

Centroidal distance for larger circle, $x_1 = 0$

Centroidal distance for smaller circle, $x_2 = \frac{r/2}{2} = \frac{r}{4}$

Centroidal distance, $\bar{x} = \frac{A_1 \bar{x}_1 \; A_2 \bar{x}_2}{A_1 \; A_2}$

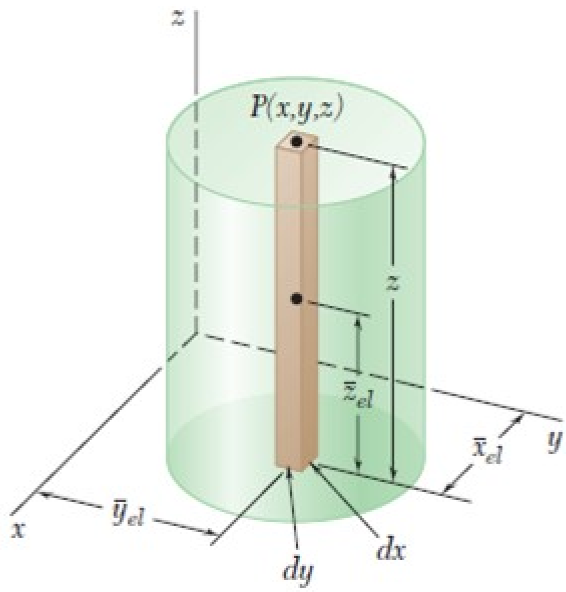
Centroidal distance from 1-1 axis,

$$\bar{y} = \frac{ay}{a}$$

$$= \frac{8.810^3}{541.68}$$

$\bar{y}$ **= 15.11 cm** **(Ans)**

DETERMINATION OF CENTROIDS OF VOLUMES BY INTEGRATION



$\bar{x}_{el} = x,\ \bar{y}_{el} = y,\ \bar{z}_{el} = \frac{z}{2}$
$dV = z\,dx\,dy$

Determination of the centroid of a volume by double integration.

The centroid of a volume bounded by analytical surfaces can be determined by evaluating the integrals given

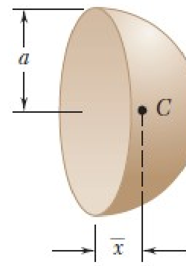
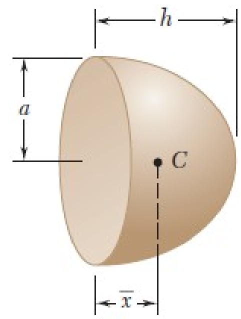
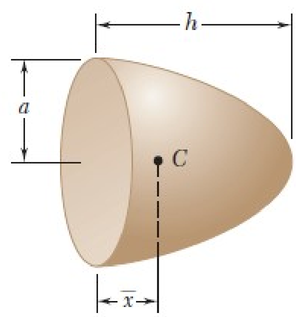
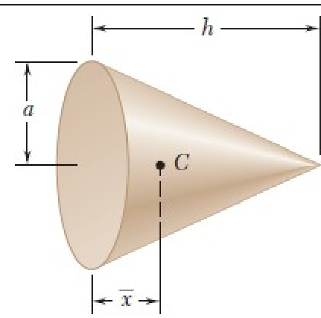
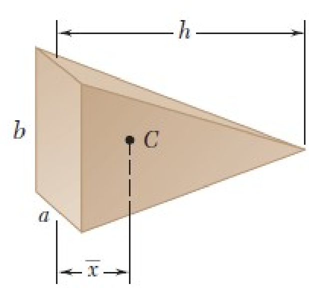
$$\bar{x}V = \int x\,dV \qquad \bar{y}V = \int y\,dV \qquad \bar{z}V = \int z\,dV$$

If the element of volume dV is chosen to be equal to a small cube of sides dx, dy, and dz, the evaluation of each of these integrals requires a *triple integration*. However, it is possible to determine the coordinates of the centroid of most volumes by *double integration* if dV is chosen to be equal to the volume of a thin filament
The coordinates of the centroid of the volume are then obtained by rewriting Eqs. as

$$\bar{x}V = \int \bar{x}_{el}\,dV \qquad \bar{y}V = \int \bar{y}_{el}\,dV \qquad \bar{z}V = \int \bar{z}_{el}\,dV$$

Theorem of papus

The volume of body of revolution obtained by revolving area is equal to the product of the generating area and the distance travelled by the centroid of the generating area, while the body is being generated.

Shape		$\bar{x}$	Volume
Hemisphere		$\frac{3a}{8}$	$\frac{2}{3}\pi a^3$
Semiellipsoid of revolution		$\frac{3h}{8}$	$\frac{2}{3}\pi a^2 h$
Paraboloid of revolution		$\frac{h}{3}$	$\frac{1}{2}\pi a^2 h$
Cone		$\frac{h}{4}$	$\frac{1}{3}\pi a^2 h$
Pyramid		$\frac{h}{4}$	$\frac{1}{3}abh$

PROOF :

Let the lamina shown in fig of area A is revolved about x- axis through the angle of 2 radians. Consider an elemental area dA which is located at a distance of y from x- axis

The element of volume obtained by revolving the area dA is dV = 2 y d **A**

The volume generated by the entire area is

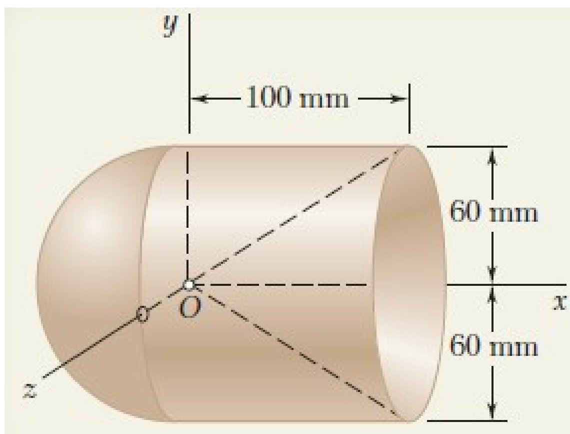
V= dv 2 ydA 2 ydA

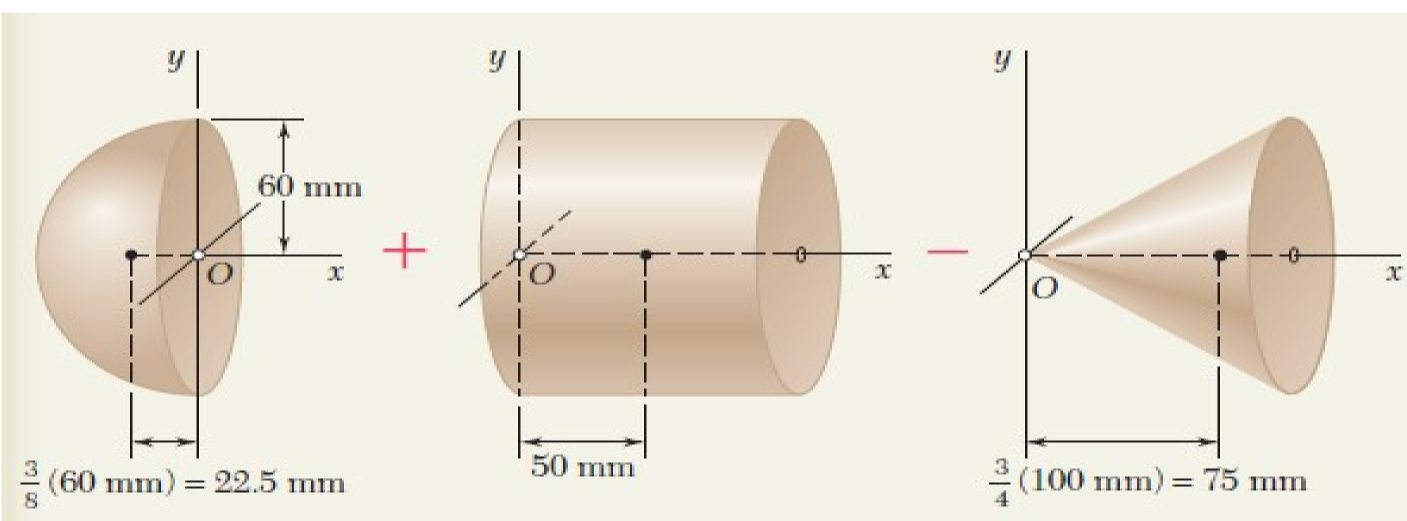
But ydA yA

V= 2 yA_

Here, the term 2 yA is the distance travelled by the centroid of the area.

Determine the location of the center of gravity of the homogeneous body of revolution shown, which was obtained by joining a hemisphere and a cylinder and carving out a cone.





Component	Volume, mm^3	$\bar{x}$, mm	$\bar{x}V$, mm^4
Hemisphere	$\frac{1}{2}\frac{4\pi}{3}(60)^3 = 0.4524 \times 10^6$	-22.5	-10.18×10^6
Cylinder	$\pi(60)^2(100) = 1.1310 \times 10^6$	$+50$	$+56.55 \times 10^6$
Cone	$-\frac{\pi}{3}(60)^2(100) = -0.3770 \times 10^6$	$+75$	-28.28×10^6
	$\Sigma V = 1.206 \times 10^6$		$\Sigma \bar{x}V = +18.09 \times 10^6$

Thus,

$$\bar{X}\Sigma V = \Sigma \bar{x} V: \qquad \bar{X}(1.206 \times 10^6 \text{ mm}^3) = 18.09 \times 10^6 \text{ mm}^4$$

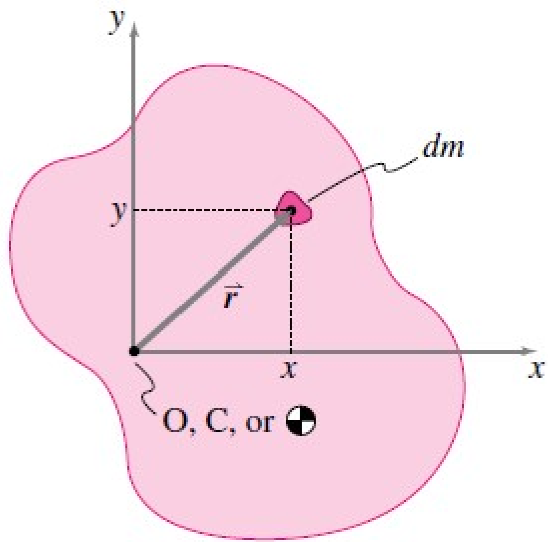
$$\bar{X} = 15 \text{ mm}$$

MOMENT OF INERTIA

The concept of Inertia is provided by Newton's. I law. Inertia is the property of the matter by virtue of which it resists any change in its state of rest or of uniform motion.

Polar moment of inertia

In particular, all of the terms in all of momentum, angular momentum, and energy equations concern sums over all the bits of mass in a system, with each bit of mass multiplied by some terms concerning position, velocity and acceleration. From the earlier sections in this chapter we know how to find the velocity and acceleration of every bit of mass on a 2-D rigid body as it spins about a fixed axis. So it is just a matter of doing integrals or sums to calculate the various momentum and energy quantities of interest. As a body moves and rotates the region of integration and the values of the integrands change. So, in principle, in order to analyze a rigid body one has to evaluate a different integral or sum at every different configuration. But there is a shortcut: for a rotating rigid object a sum (over all atoms, say), or a difficult integral (for example, over the complex region representing a machine part) is reduced to simple multiplication.



The *moment of inertia* I^{cm}(1) simplifies the expressions for the angular momentum, the rate of change of angular momentum, and the energy of a rigid body. For more general motions the shortcuts need a 3×3 matrix $[\boldsymbol{I}^{\text{cm}}]$ But for 2D mechanics only one component of the matrix $[\boldsymbol{I}^{\text{cm}}]$ is relevant, it is I_{zz}^{cm}, called just I or J for short.

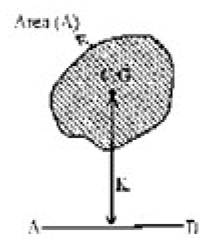
RADIUS OF GYRATION

The moment of Inertia of an area is a measure of the distribution of the area from the axis. If the whole of area of the body shown in fig. is assumed to be concentrated at a distance k from AB, then

$$I_{AB} = AK_{AB}^2 \qquad \text{....... (1)}$$

This distance K_{AB} is called as radius of gyration

Thus, the radius of gyration is defined as the distance at which the whole area of the body may be assumed to be concentrated with reference to the axis of reference.



Thus

$$K_{AB} = \sqrt{\frac{I_{AB}}{A}}$$

In general,

Radius of gyration with respect to y - axis,

$$K_{yy} = \sqrt{\frac{I_{yy}}{A}}$$

Radius of gyration with respect to x-axis,

$$K_{xx} = \sqrt{\frac{I_{xx}}{A}}$$

PARALLEL AXES THEOREM

Statement :

Parallel axis theorem states that, " The moment of inertia of a plane area about any axis is the sum of the moment of inertia of the area about the axis, passing through the centroid of area parallel to the given axis and the product of area of the plane and the square of the perpendicular distance of its centroid from the axis"

PERPENDICULAR AXES THEOREM

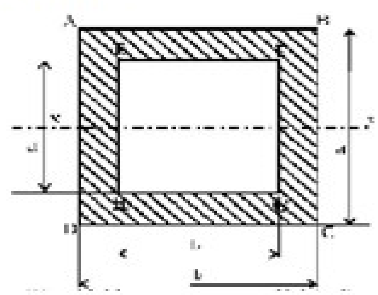
Statement :

Perpendicular axes theorem states that, "If I_{xx} and I_{yy} be the moments of inertia of a plane lamina about two mutually perpendicular axes OX and OY meeting at O, and moment of inertia I_{zz} about the axis z-z, perpendicular to the plane and passing through the intersection of axes x-x and y-y is given by the relation

$$I_{zz} = I_{xx} + I_{yy}$$

Moment of Inertia of Hollow Rectangular Section :

Consider a hollow rectangular section in which ABCD is the main section and EFGH is the cut out section as shown in



Let

b = breadth of outer rectangle

d = depth of outer rectangle

b_1= breath of cut out rectangle

d_1= depth of cut out rectangle

Moment of Inertia of hollow rectangular section about x-x axis

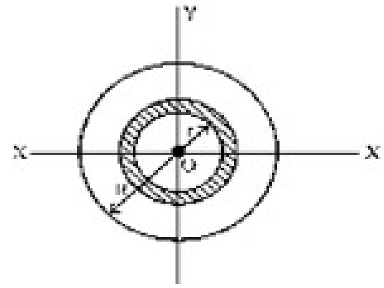
I_{x-x} = M.I of rectangle ABCD - M.I. of rectangle EFGH

$$\boxed{I_{x-x} = \frac{bd^3}{12} \quad \frac{b\,d^3}{12}}$$

similarly

$$\boxed{I_{y-y} = \frac{db^3}{12} \quad \frac{d\,b^3}{12}}$$

Moment of Inertia of Circular Section :



Consider a circular lamina of radius R. The lamina may be considered as consisting of elemental concentric rings. Consider one such elemental ring at a radius ‘r’ and having a thickness ‘dr’

Let x-x be the horizontal axis passing through the Centroid of the Circular section.

Area of elementary ring, dA= 2 rdr

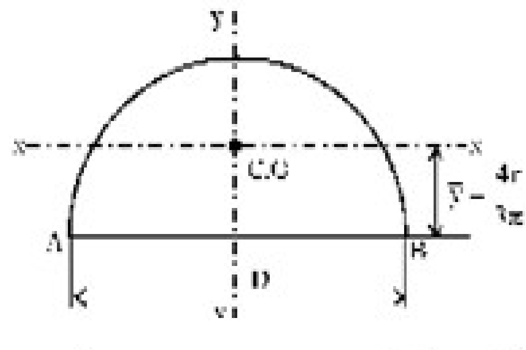
Moment of inertia of elementary ring about polar axis z-z through O. Perpendicular to x-y plane

$$d\,I_{z\text{-}z} = \text{Area} \times (\text{distance})^2$$

$$= dA \times r^2$$

Moment of Inertia of a Semi-Circle :

Consider a semi circular section of radius ‘R’ as shown



Case (i) : M.I about its diameter AB :

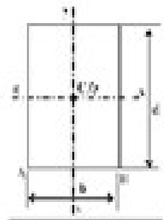
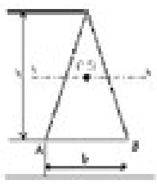
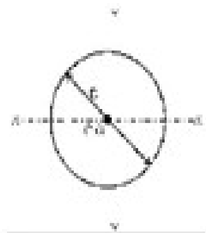
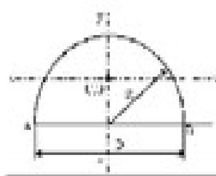
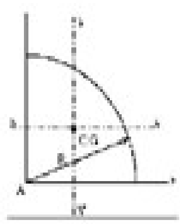
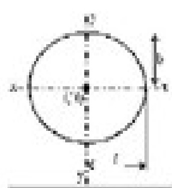
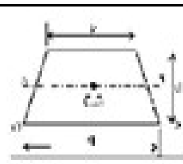
Moment of inertia of semi circle about its diameter i.e., about AB

$$= \frac{\text{Moment of inertia of circle about AB}}{2}$$

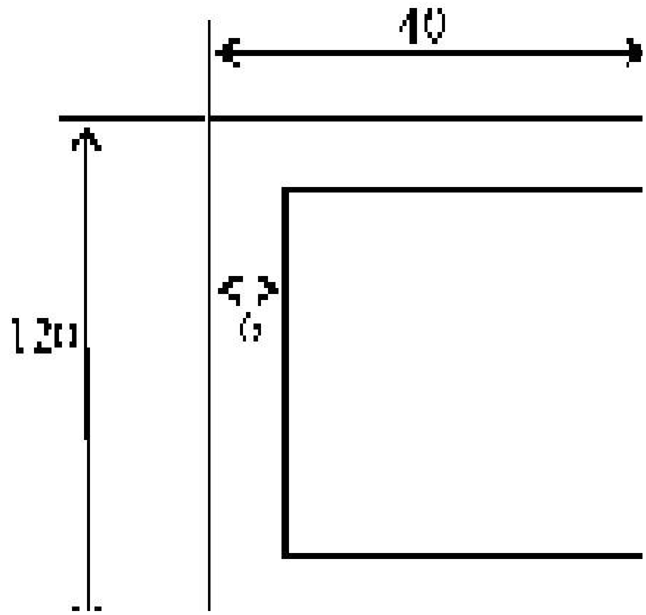
$$I_{AB} = \frac{D^4 / 64}{2}$$

$$I_{AB} = \frac{D^4}{128}$$

MI of common shapes

SHAPE	FIGURE	AREA	MOMENT OF INERTIA		
			About Base, I_{AB}	About x-x axis, $I_{x\text{-}x}$	About y-y axis $I_{y\text{-}y}$
Rectangle		b.d	$I_{AB} = \frac{bd^3}{3}$	$I_{xx} = \frac{bh^3}{12}$	$I_{yy} = \frac{bh^3}{12}$
Triangle		$\frac{1}{2}$ b.h	$I_{AB} = \frac{bh^3}{12}$	I_{xx} $\frac{bh^3}{36}$	- -
Circle		$\frac{\pi D^2}{4}$	-	$I_{xx} = \frac{\pi D^4}{64}$	$I_{yy} = \frac{\pi D^4}{64}$
Semi-Circle		$\frac{\pi D^2}{8}$	$I_{AB} = \frac{\pi D^4}{128}$	$I_{xx} = 0.11R^4$	$I_{yy} = \frac{\pi D^4}{64}$
Quarter		$\frac{D^2}{16}$	$I_{AB} = \frac{\pi D^4}{256}$	$I_{xx} = 0.055R^4$	$I_{yy} = \frac{\pi D^4}{256}$
Ellipse		ab	-	I_{xx} $\frac{\pi ab^3}{4}$	I_{yy} $\frac{\pi a^3 b}{4}$
Trapezium		$\frac{P+q}{2}\cdot$	-	I_{xx} $\frac{P^2+4Pq+q^2}{36(P+q)} \times d^3$	-

Find the moment of Inertia of a channel section shown below



Moment of inertia of the given section about x-x axis

$I_{x\text{-}x}$= M.I of Rectangle (1) about x-x axis + M.I of rectangle (2) about x-x axis + M.I of Rectangle (3) about x-x axis

$$= [I_{self1} + a_1 (y_1 - \bar{y})^2] + [I_{self2} + a_2(\bar{y} - y_2)^2] + [I_{self3} + a_3(\bar{y} - y_3)^2]$$

$$=[1706.66+320\,(116-60)^2] + [562432+60\,(60-60)^2] + [1706.66+320\,(60-4)^2]$$

$$= 1005226.66 + 562432 + 1005226.66$$

$I_{x\text{-}x}$ = 2572885.32 mm^4 **(Ans)**

M.I of section about y-y axis, I-yy:

Component	Area a (mm^2)	Centroidal Distance from 2-2 axis 'x' (mm)	ax (mm^3)	I self about the axis y-y (mm^4)
Rectangle	a_1 = 40x8 = 320	$x_1 = \frac{40}{2} = 20$ = 116	6400	$I_{self_1} = \frac{d_1 b_1^3}{12} = \frac{40^3\ 8}{12}$ = 42666.67
Rectangle	a_2 = 104x6 = 624	$x_2 = \frac{6}{2} = 3$	1872	$I_{self_2} = \frac{d_2 b_2^3}{12} = \frac{104\ 6^3}{12}$ = 1872
Rectangle	a_3 = 40x8 = 320	$x_3 = \frac{40}{2} = 20$	6400	$I_{self_3} = \frac{d_3 b_3^3}{12}$ $\frac{8\ 40^3}{12}$ = 42666.67
	a = 1264		ay =14672	

Distancy of centroidal axis y-y from 2-2 axis

$$\bar{x} = \frac{ax}{a} = \frac{14672}{1264} = 11.61mm$$

Component	Area a (mm^2)	Centroidal Distance from 1-1 axis 'y' (cm)	ay (mm^3)	I self about the axis x-x (cm^4)
Rectangle	$a_1 = 6x3$ $= 18$	$y_1 = \frac{3}{2} + 13 + 4$ $y_1 = 18.5$	333	$I_{self_1} = \frac{\ ^3}{12}$ $= \frac{6\ 3^3}{12}$ $= 13.5$
Rectangle	$a_2 = 13x2$ $= 26$	$y_2 = \frac{13}{2} + 4$ 10.5	273	$I_{self_2} = \frac{b_2 d_2^3}{12}$ $= \frac{2\ 13^3}{12}$ $= 366.167$
Rectangle	$a_3 = 10x4$ $= 40$	$y_3 = \frac{4}{2} = 2$	80	$I_{self_3} = \frac{b_3 d_3^3}{12}$ $\frac{10\ 4^3}{12} = 53.34$
	a = 84		ay = 686	

Distancy of centroidal axis x-x from axis 1-1

$$\bar{y} = \frac{ay}{a} = \frac{686}{84} = 8.17cm$$

M.I of section about x-x axis

I_{x-x} = M.I of rectangle (1) about x-x axis +
M.I of rectangle (2) about x-x axis +
M.I of rectangle (3) about x-x axis

M.I of section about x-x axis

I_{x-x} = M.I of Rectangle (1) + M.I of Rectangle (2) + M.I of Rectangle (3)

$= [I_{self1} + a_1(y_1 - \bar{y})^2] + [I_{self\ 2} + a_2(\bar{y} - y_2)^2] + [I_{self3} + a_3(y_3 - \bar{y})^2]$

A $[36+12\ (5-2.8)^2] + [6.67+20(2.8-1)^2] + [10.67+8\ (4-2.8)^2]$

B $94.08 + 71.47 + 22.19$

$I_{x-x} = 187.74.\ cm^4$ **(Ans)**

To Find M.I about y-y axis, I_{y-y} :

Component	Area a (cm^2)	Centroidal Distance from 2-2 axis 'x' (cm)	ax (cm^3)	I self about the axis y-y (cm^4)
Rectangle	$a_1 = 6 \times 2 = 12$	$x_1 = \frac{2}{2} = 1$	12	$I_{self_1} = \frac{b_1 d_1^3}{12} = \frac{6 \times 2^3}{12} = 4$
Rectangle	$a_2 = 10 \times 2 = 20$	$x_2 = \frac{10}{2} = 5$	100	$I_{self_2} = \frac{b_2 d_2^3}{12} = \frac{2 \times 10^3}{12} = 166.67$
Rectangle	$a_3 = 4 \times 2 = 8$	$x_3 = \frac{2}{2} + 8 = 9$	72	$I_{self_3} = \frac{b_3 d_3^3}{12} = \frac{4 \times 2^3}{12} = 2.67$
	$a = 40$		$ax = 184$	

Distance of centroidal axis y-y from 2-2 axis

$$\bar{x} = \frac{ax}{a} = \frac{184}{40} = 4.6$$

M.I about y-y axis is

$$I_{y-y} = \frac{db^3}{12} - \frac{d_1 b_1^3}{12}$$

$$= \frac{20 \ (15)^3}{=} - \frac{10 \ (8)^3}{12}$$

$$= 5625 - 426.67$$

$$\mathbf{I_{y-y} = 5198.34\ cm^4} \qquad \textbf{(Ans)}$$

To find radius of gyration about the base:

M.I of base AB is

$$I_{AB} = \frac{bd^3}{3} - \frac{b_1 d_1^3}{3}$$

$$= \frac{15 \ (20)^3}{3} - \frac{8 \ (1}{}$$

$$= 40000 - 2666.67$$

$$I_{AB} = 37333.34 cm^4$$

Radius gyration about base AB, K_{AB} =

$$= \sqrt{\frac{I_{AB}}{(bd \ b_1 d_1)}}$$

37333.34

(15 20 8 10

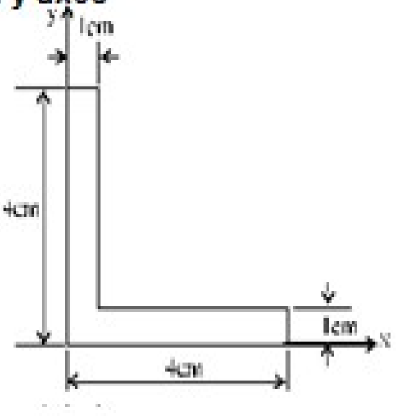
K_{AB} = 13.026 cm (A

Product of Inertia :

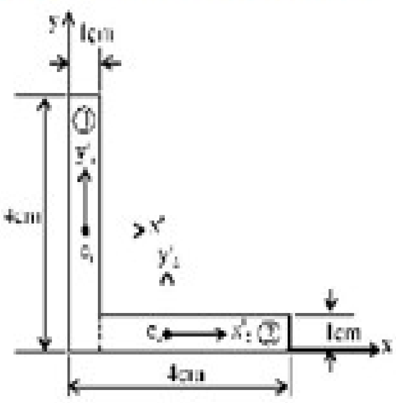
The sign of product of inertia depends upon the Co-ordinates of the various small areas of the plane figure (with reference to the co-ordinate axes xx and yy about which the product of inertia is to be found out)

Find the product of inertia of L -section shown in fig. with respect to x and y axes

Solution :



Split the given area in two rectangles viz I and 2 as shown in fig



(1) Rectangle (2) Triangle (3) Semi-Circle

Rectangle (1) :

$$I_{x_1y_1} = I_{x_1y_1} + A\,\bar{x}_1\bar{y}_1$$

$$= 0 + (40 \times 60)\ \frac{40}{2}\ \frac{60}{2}$$

[since x_1 & y_1 are axes of symmetry $I_{x_1y_1} = 0$]

$$= 2400 \times 20 \times 30$$

$$I_{x_1y_1} = 1440000\ \text{mm}^4$$

Triangle (2) :

By parallel axis theorem,

$$I_{x2y2} = I_{x_2y_2} + A\,\bar{x}_2\bar{y}_2$$

$$= \frac{b^2h^2}{72} + \frac{1}{2}bh\ 40 - \frac{6h}{3}\ 3$$

$$= -\frac{(30)^2\ (60)^2}{72} + \frac{1}{2} \times 30 \times 60\ 40\ \frac{30}{3}\ \frac{60}{3}$$

$$= -45000 + 900000$$

$$I_{x2y2} = 855000\text{mm}^4 \qquad \textbf{(Ans)}$$

Semi-Circle (3) :

By parallel axis theorem,

$$I_{x3y3} = I_{x_3y_3} + A\,\bar{x}_3\bar{y}_3$$

$$= 0 + \frac{2}{2}\ 10\frac{4\ 20}{3}(30)$$

$$= 200\ 300\ \frac{800}{}$$

$$I_{x3y3} = 348500\text{mm}^4 \qquad \textbf{(Ans)}$$

$I_{xy} = [I\, x_1 y_1 + a_1 \bar{x}_1 \bar{y}_1] + [I\, x_2 y_2 + a_2 \bar{x}_2 \bar{y}_2] + [I\, x_3 y_3 + a_3 \bar{x}_3 \bar{y}_3]$

Since $x_1, x_2, x_3, y_1, y_2, y_3$ are the axes of symmetry

$I_{x_1 y_1} = 0, I_{x_2 y_2} = 0, I_{x_3 y_3} = 0$

$I_{xy} = [0 + (-810) + (0 + 0 \times 0) + [0 + (-80)]$

$\mathbf{I_{xy} = -1620cm^4}$ **(Ans)**

2. To Find the Principal Axes of Section about 'o' :

M.I of given section about x-x

$I_{x\text{-}x}$ = M.I. of Rectangle (1) about x-x + M.I of Rectangle – about x-x + M.I of Rectangle (3) about x-x

$$= [I_{self1} + a_1 (y_1)^2] + \frac{b_2 d_2^3}{3} + [I_{self3} + a_3 (y_3)^2]$$

$$= \frac{12 \; (3)^3}{12} (12 \;\; 3)(4.5)^2 \qquad \frac{2 \; (6)^3}{3}$$

$$= \frac{12 \; (3)^3}{12} (12 \;\; 3)(4.5)^2$$

$= 756 + 144 + 756$

$I_{x\text{-}x} = 1656 \text{ cm}^4$

$$I_{y\text{-}y} = [I_{self1} + a_1 (x_1)^2] + \frac{b_2^3 d_2}{3} + [I_{self3} + a_3 (x_3)^2]$$

$$= \frac{3 \; (12)^3}{12} \quad (12 \;\; 3)(5)^2 + \frac{(2)^3 \;\; 6}{3} + \frac{3 \; (12)^3}{12} (12 \;\; 3)(5)^2$$

$= 1332 + 16 + 1332$

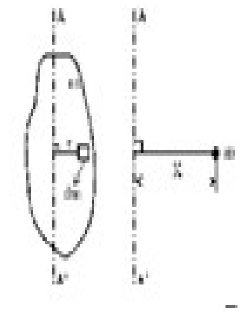
$I_{y\text{-}y} = 2680 \text{cm}^4$

MASS MOMENT OF INERTIA

Definition :

The mass moment of inertia of system is a measure of the inertia of the system.i.
is a resistance offered by the system to the rotational acceleration of the mass of body

Consider a three dimensional body of mass 'm' as shown in fig.



Consider an elemental mass 'dm' at a distance of 'r' from the axis AA'

The mass moment of inertia about the axes AA' is defined as

$$I_{AA} = r^2 dm$$

Radius of Gyration (k) of Mass :

The radius of gyration 'k' of the body with respect to the axis AA' is given b
the relation,

$$I = K^2 m$$

$$K = \sqrt{\frac{I}{M}}$$

where,

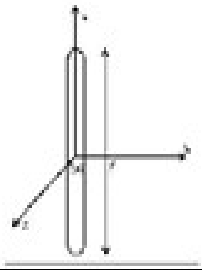
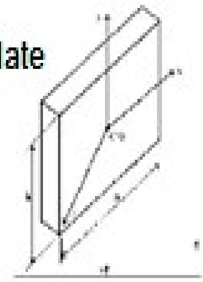
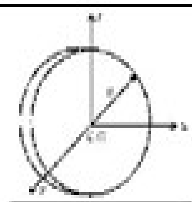
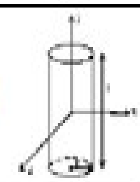
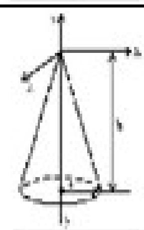
I = mass moment of inertia of the body

m = mass of the body.

Unit of Mass Moment of Inertia :

The unit of moment of Inertia of mass kg.m^3, when mass is expressed in kilograr
and radius of gyration in meters.

MASS MOMENT OF INERTIA OF VARIOUS BODIES

SHAPE FIGURE	MASS	MASS MOMENT OF INERTIA		
		I_{x-x}	I_{y-y}	I_{z-z}
Slender Rod 	M	$\frac{Mp^2}{12}$	0	$\frac{Mp^2}{12}$
Rectangular Plate 	M = bht	$\frac{Mh^2}{12}$	$\frac{Mh^2}{12}$	$\frac{M}{12}(h^2+b^2)$
Circular Plate 	$M = \pi R^2 tp$	$\frac{MR^2}{4}$	$\frac{MR^2}{4}$	$\frac{MR^2}{2}$
Circular Cylinder 	$M = \pi R^2 hp$	$\frac{M}{12}(3R^2+h^2)$	$\frac{MR^2}{2}$	$\frac{M}{12}(3R^2+h^2)$
Circular Cone 	$M = \frac{1}{3}\pi R^2 hp$	$\frac{R^2}{4}+h^2$	$\frac{3}{10}MR^2$	$\frac{3M}{5}\left(\frac{R^2}{4}+h^2\right)$

UNIT IV

FRICTION

FRICTION

A force which prevents the motion or movement of the body is called friction or force of friction and its direction is opposite to the applied external force or motion of the
body. Friction is a force of resistance acting on a body which prevents or retards motion of the body. Or When a body slides upon another body, the property due to which the motion of one relative to the other is retarded is called friction. This force always acts tangent to the surface at points of contact with other body and is directed opposite to the motion of the body.

1. Static Friction :
The static friction is the friction experienced by a body, when it is at rest. In other words, it is the friction when the body tends to move.

2. Dynamic Friction (or) Kinetic Friction :
The dynamic friction is the friction experienced by a body when it is in motion. Dynamic friction is always less than static friction. It is about 40 to 75 percent of limiting static friction. Dynamic friction is again divided in two types namely,

(a) Sliding Friction :
The friction that exists when one body slides over the other is called sliding friction. Example : A piston moving in the cylinder of an engine.

(b) Rolling Friction :
The friction experienced by a body when it rolls over the other body is called Rolling friction. In other words, it is the friction occurs, when two surfaces are separated by balls or rollers.
Example : Wheel or cylinder rolling over a surface. Rolling friction is always less than the sliding friction.

Friction in Lubricated Surface :
When a lubricating fluid is introduced between the contact surface of two bodies, fluid friction is developed. It may be of following two types.

1. Nonviscous or Boundary Friction :

If in between two rubbing surfaces, there exists a thin film or layer of an oil or lubricant, the oil gets absorbed in the surfaces. Thus, there is no metal to metal contact of surface but there is a contact between thin layer if the oil and obviously the frictional force is reduced. In such a case, the frictional force is known as boundary friction.

2. Viscous or Film Friction :

When the two surfaces are completely separated by a thick layer of lubricant or a fluid, then, This limiting stage, when the block just start to move, is the ending motion stage. The zone up to impending motion is is said to be range of static friction.

CO-EFFICIENT OF FRICTION

It is defined as the ratio of limiting force of friction (F) to the normal reaction (R) It s denoted by μ

$$\mu = \frac{\text{Limiting force of friction}}{\text{Normal reaction}} = \frac{F}{N}$$

$$\mathbf{F = \mu N}$$

Co-efficient of friction is a measure of degree of roughness between the contact surfaces.

As 'μ' is a pure ratio, it has no units.

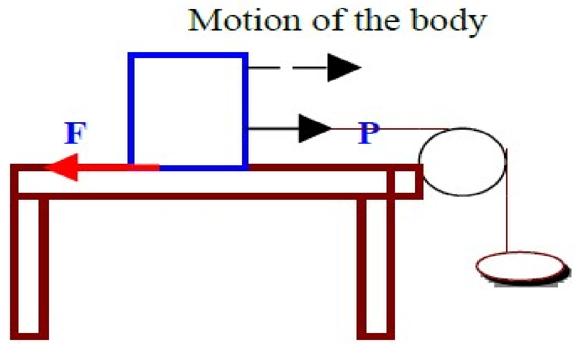
ANGLE OF REPOSE

It is the maximum inclination of a plane with the horizontal at which a body is just begins to slide down the plane. In other words, the maximum inclination of the plane on which a body, free from external forces, can repose (steep) is called angle of repose.

LIMITING FRICTION

The maximum friction (before the movement of body) which can be produced by the surfaces in contact is known as limiting friction

It is experimentally found that friction directly varies as the applied force until the movement produces in the body. Let us try to slide a body of weight w over another body by a force P as shown in fig



LAWS OF FRICTION

These laws are listed below:

1. Laws of Static Friction

1 The force of friction always acts in a direction opposite to that in which the body tends to move.
2 The magnitude of force of static friction is just sufficient to prevent a body from moving and it is equal to the applied force.
3. The force of static friction does not depend upon, shape, area, volume, size etc. as long as normal reaction remains the same.
4. The limiting force of friction bears a constant ratio to normal reaction and this constant ratio is called coefficient of static friction.

2. Laws of Dynamic Friction

1. The force of friction always acts in a direction, opposite to that in which the body is moving
2. The magnitude of friction force is less than that of applied force.
3. The magnitude of force of dynamic friction bears a constant ratio to the normal reaction (N) between the two surfaces. But this ratio is slightly less than incase of limiting friction.
4. The frictional force remains constant for moderate speeds but it decreases slightly with the increase of speed.

A body weight 50N rests on a rough, horizontal surface. How much h force is necessary just to move it? The static co-efficient of friction the body and the surface is 0.1. What horizontal force is necessary the body moving if the co-effiecient of dynamic friction be 0.08?

Given :

Weight of body, $W = 50N$

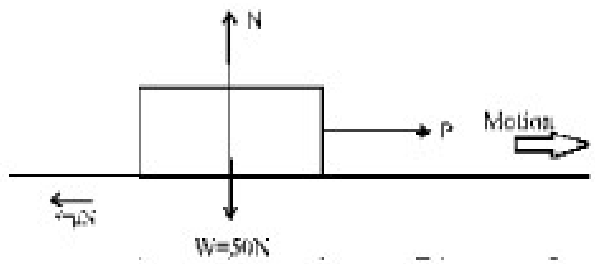
Co-efficient of static friction, $\mu_S = 0.1$

Co-efficient of Dynamic friction, $\mu_k = 0.08$

TO FIND

Applied force P = ?

Solution :



Resolving forces horizontally,

$$\Sigma F_x = 0$$

$$P - F = 0$$

$$P = \mu N \qquad ....(1)$$

Resolving forces vertically,

$$\Sigma F_y = 0$$

$$N = W$$

$$N = 50N$$

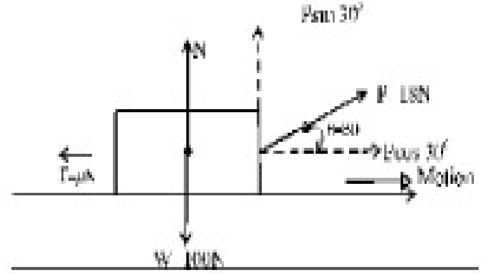
$\therefore$ (1) becomes

$$P = 50\,\mu \qquad ....(2)$$

Solution :

Case:1 when the body is pulled.

The body is in equilibrium under the action of forces shown in fig



Resolving the forces horizontally

$$\Sigma F_x = 0;$$

$$P\cos 30^0 - F = 0$$

$$18\cos 30^0 = \mu N$$

$$\mu N = 15.59 \qquad \text{..............(1)}$$

Resolving the forces vertically

$$\Sigma F_y = 0;$$

$$N + P\sin 30^0 - W = 0$$

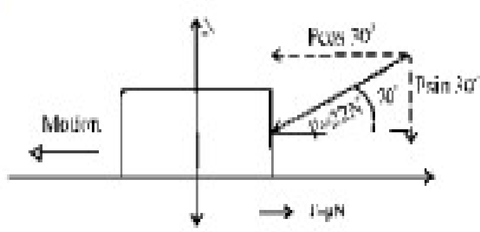
$$N = W - 9 \qquad \text{..............(2)}$$

Put (2) in (1) we get,

$$\mu(W-9) = 15.59 \qquad \text{.............. (3)}$$

Case : 2 When the body is pushed.

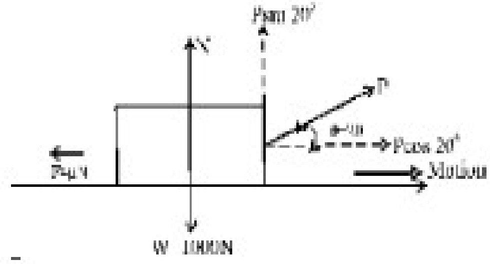
The body is in equilibrium under the action of forces shown in fig.



Force applied, P = ?

Solution :

The forces acting on the block is shown in



Resolving forces horizontally

$$\Sigma F_x = 0$$

$$P \cos 20^0 - \mu N = 0$$

$$P \cos 20^0 = 0.6\ N \qquad \text{.............(1)}$$

Resolving forces vertically

$$\Sigma F_y = 0$$

$$N + P \sin 20^0 - W = 0$$

$$N = 1000 - P \sin 20^0$$

Substituting the value of 'N' in (1) we get

$$P \cos 20^0 = 0.6\ (1000 - P\sin 20)$$

$$P \cos 20^0 = 600 - 0.6\ P\sin 20^0$$

$$P\ (\cos 20^0 + 0.6 \sin 20^0) = 600$$

$$P = \frac{600}{(\cos 20^\circ + 0.6 \sin 20^\circ)} \quad \text{.....(2)}$$

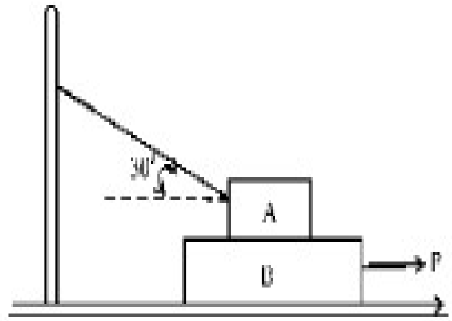
$$P = \frac{600}{(0.9397 + 0.6 \times 0.342)}$$

$$P = \frac{600}{1.1449}$$

$$\mathbf{P} \quad = \frac{600}{1.1661}$$

$$\mathbf{P} \quad = \mathbf{514.5N} \qquad \textbf{(Ans)}$$

Two blocks A and B of weight 1kN and 2kN respectively are in equilibrium as shown in fig. 6.13. If the co-efficient of friction between the two blocks as well as the block B and the floor is 0.3. Find the force 'P' required to move the block B, when i) the string is tied to block A tightly ii) the string is removed.



Given :

Weight of block A, $W_A = 1kN$

Weight of block B, $W_B = 2kN$

Co-efficient of friction, $\mu = 0.3$

TO FIND

Force 'P' for given conditions.

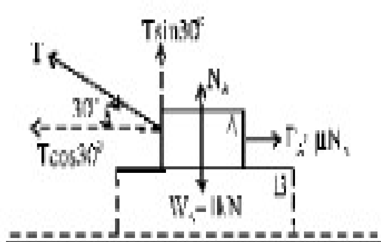
Solution :

Let 'T' be the tension in the string.

(i) When string is tied to block A :

Equilibrium of block 'A' :

The forces acting on block 'A' is shown in fig.



Resloving forces vertically, we get

$$N_B - N_A - w_B = 0$$

$$N_B - 0.85 - 2 = 0$$

$$N_B = 2.85$$

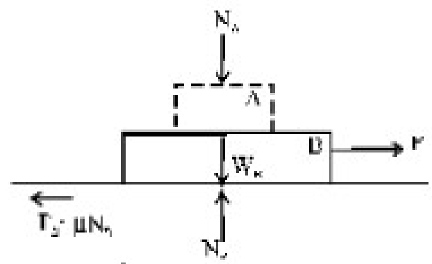
Put $N_B = 2.85$ in (2) we get

$$P = 0.3\ (0.85 + 2.85)$$

P = 1.11kN. **(Ans)**

2. When the string is removed :

The forces acting on block 'A' and block 'B' when the string is removed is shown in fig.



Resolving forces horizontally,

$$\Sigma F_x = 0;$$

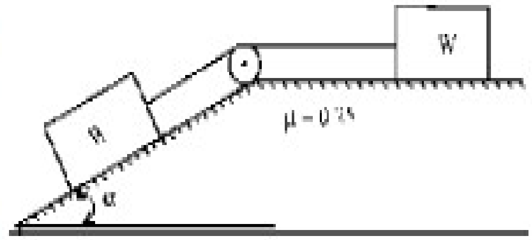
$$P - F_B = 0$$

$$P = \mu N_B$$

$$P = 0.3\ (2.85)$$

P = 0.855 kN **(Ans)**

Two blocks of equal weights 'W' rest on two surfaces of same co-efficient of static friction, $\mu = 0.25$ as shown in fig.

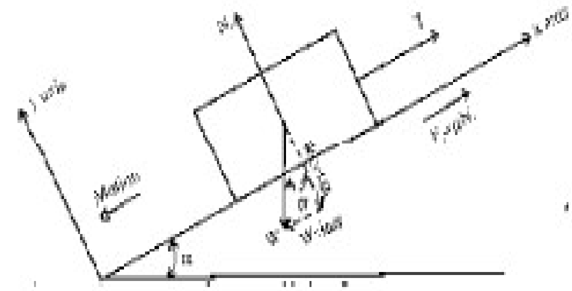


The blocks are connected by rope passing over a frictionless pulley. Find, for what value of a, the motion of two blocks will impend?

Solution :

Let 'T' be the tension in the string

Equilibrium of Lower Block:



Resolving forces along the plane,

$$\Sigma F_x = 0 ;$$

$$T + \mu N_1 - w \sin \alpha = 0$$

$$T + 0.25N_1 = w \sin \alpha \qquad ....(1)$$

Resolving forces perpendicular to the plane,

$$\Sigma F_y = 0 ;$$

$$N_1 - w \cos \alpha = 0$$

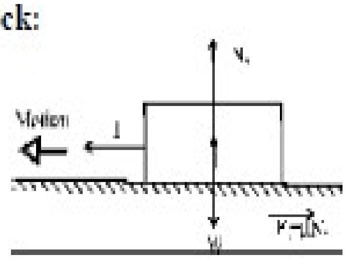
$$N_1 = w \cos \alpha$$

Substituting value of 'N' in equation (1) weget,

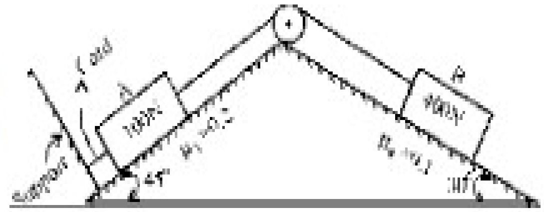
$$T + 0.25 (w \cos \alpha) = W \sin \alpha$$

$$T = W (\sin \alpha - 0.25 \cos \alpha) \qquad ....(2)$$

Equilibrium of Upper Block:



Find the maximum tension in the cord shown in fig. if the bodies have developed full friction.



Given :

Weight of block A, W_A = 100N

weight of block B, W_B = 400N

Co-efficient of friction of block A, μ_A = 0.2

Co-efficient of friction of block B, μ_B = 0.1

TO FIND

Maximum Tension in cord = ?

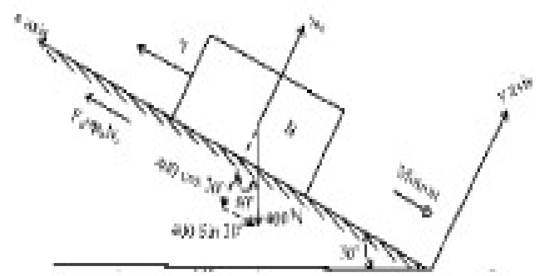
Solution :

Let T = Tension in the cord connecting 100N with 400N

T_1 = Tension in the cord connecting 100N with support

Equilibrium of Block, B :

The forces acting on block 'B' is shown in fig.



Resolving forces along the plane

$$\Sigma F_x = 0$$

$$T + \mu_B N_B - 400 \sin 30^0 = 0$$

$$T + 0.1N_B - 200 = 0$$

$$T = 200 - 0.1N_B \qquad \text{..........(1)}$$

Resolving forces perpendicular to the plane,

$$\Sigma F_y = 0$$

$$N_B = 400 \cos 30^0 = 0$$

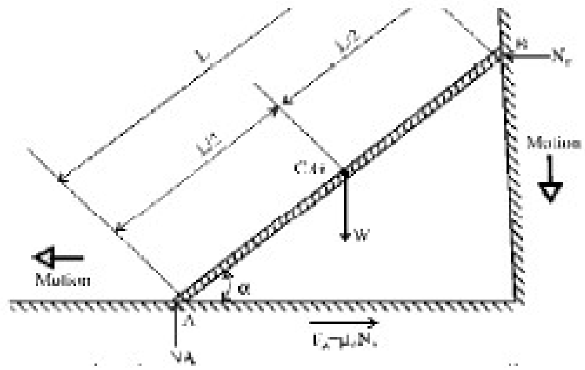
$$N_B = 346.4 \text{ N}$$

Substituting this value of N_B in eqn. (1), we get

$$T = 200 - 0.1\,(346.4)$$

$$\mathbf{T = 165.36N} \qquad \text{(Ans)}$$

Ladder Friction



Due to the self weight of ladder, the upper end (B) of the ladder tends downwards and hence the force of friction between the ladder and the wall (F_B= will be acting upwords.

Similarly, the lower end A of the ladder will tend to move towards right, the friction ($F_A = \mu_A N_A$) will be acting towards left.

For the equilibrium of ladder,

(i) The algebric sum of horizontal components of the forces must be zero i.e, ΣH

(ii) The algebric sum of vertical components of the forces must be zero i.e, Σ

(i) The algebric sum of horizontal components of the forces must be zero i.e, $\Sigma H = 0$.

(ii) The algebric sum of vertical components of the forces must be zero i.e, $\Sigma V = 0$.

(iii) The algebric sum of moments of all forces about any point (A or B) must be zero i.e $\Sigma M = 0$.

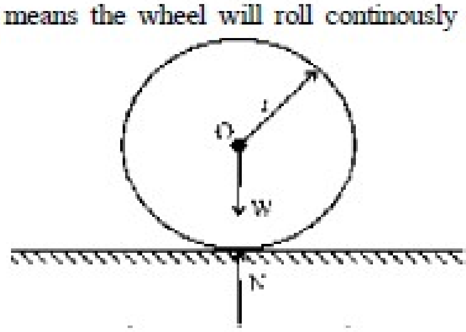
ROLLING RESISTANCE

Consider a wheel which rolls over a horizontal surface at constant speed as shown in The forces acting on the wheel are

(i) Self -weight (W) of wheel

(ii) normal reaction (N) of ground.

So, here there is no friction force. This means the wheel will roll continously and indefinitely.



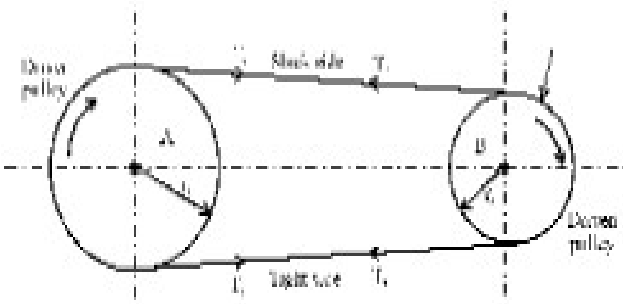
But actually, the wheel stops after sometime. This is due to the fact that the wheel and the ground deform and the contact between the wheel and the ground is not at one point as originally visualise but over an area. This contact over an area gives rise to a type of resistance which is known as rolling resistance (or) wheel friction.

BELT FRICTION

The belts or ropes are used to transmit power from one shaft to another by means of pulleys which rotate at same speed or at different speeds. Transmission of power is due to the frictional resistance developed between belt and the driving or resisting surface with which the belt is in contact.

POWER TRANSMITTED BY THE BELT

Fig shows the driving pulley 'A' and driven pulley 'B'. The driving pulley pulls the belt from one side and delivers the same to otherside. It is thus obvious that the tension on former side (i.e., tight side) will be greater than the latter side (i.e., slack side)



Let,

T_1 and T_2= Tensions in tight and slack side of the belt respectively.

r_1 and r_2 = radii of driver and follower respectively

V = velocity of the belt.

The effective tension or force acting at the circumference of the following is the difference between two tensions (i.e., T_1 - T_2)

work done per second = force x velocity

$$= (T_1 - T_2)\, V$$

Power transmitted $P = (T_1 - T_2)\, V$

Torque exerted on driving pulley = $(T_1 - T_2)\, r_1$ Nm

Torque exerted on driven pulley = $(T_1 - T_2)\, r_2$ Nm

Find the power transmitted by a belt running over a pulley of 600mm diameter at 200 r.p.m. The co-efficient of friction between the belt and pulley is 0.25, angle of lap 160^0 and maximum tension in belt is 2500N.

Given :

Diameter, d = 600 mm = 0.6m

Speed of pulley N = 200 r.p.m

Co-efficient of friction, μ = 0.25

Angle of lap, $\theta = 160^0 = 160 \text{ x } \dfrac{\pi}{180} = 2.793$ radians

Maximum tension, T_1 = 2500N

TO FIND

Power transmitted by belt, P = ?

Solution :

$$\text{Velocity of belt, } V = \frac{\pi d N}{60}$$

$$= \frac{\pi \times 0.6 \times 200}{60}$$

$$V = 6.284 \text{ m/s}$$

We know,

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

$$\frac{T_1}{T_2} = e^{(0.25 \text{ x } 2.793)}$$

$$\frac{2500}{T_2} = 2.01$$

$$T_2 = \frac{2500}{2.01} = 1244\text{N}$$

$$= \times 0.45 \times 20$$

$$= 4.714 \text{ m/s}$$

We know, for cross-belt drive

$$\sin \alpha = \frac{r_1 + r_2}{x}$$

$$\sin \alpha = \frac{0.225 + 0.1}{1.95}$$

$$\sin \alpha = 0.1667$$

$$\alpha = \sin^{-1}(0.1667) = 9.6^0$$

$$\text{angle contact, } \theta = 180^0 + 2\alpha$$

$$= 180^0 + 2 \text{ x } 9.6^0$$

$$= 199.2^0$$

$$= 199.2 \text{ x } \frac{\pi}{180} \text{ radians}$$

$$\theta = 3.477 \text{ radians}$$

We know,

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

$$\frac{1000}{T_2} = e^{0.25 \text{ x } 3.477}$$

$$T_2 = \frac{1000}{2.387}$$

$$T_2 = 419N$$

Power transmitted, $P = (T_1 - T_2)V$

$$= (1000 - 419)\ 4.714$$

$$= 2740W$$

$$\mathbf{P = 2.74 \text{ KW}} \quad \text{(Ans)}$$

DYNAMICS OF RIGID BODIES

TYPES OF RIGID BODY MOTION

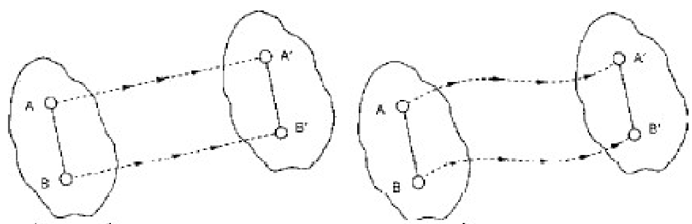
A rigid body can have the following types of motion :

(i) Translation (ii) Fixed Axis Rotation (iii) General Plane Motion

(iv) Rotation about a Fixed Point (v) General Motion

Translation

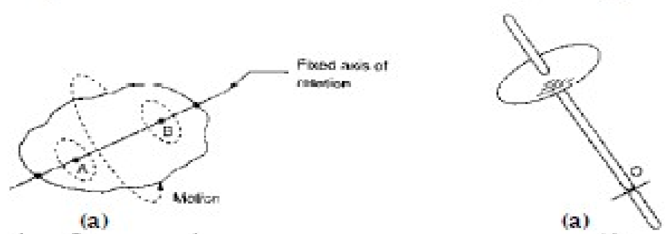
A rigid body is said to have translatory motion if an imaginary straight line drawn on the body remains parallel to the original position during its motion.



(a) Rectilinear translation **b) Curvilinear translation**

Fixed Axis Rotation

Fixed axis rotation is defined as that motion of a rigid body in which all the particles of the body, except those which lie on the axis of rotation, moving along circular paths.



(a) (a)

The planes of the circles in which the particles move are perpendicular to the axis of rotation as shown in Fig. Also the particles located on the axis have zero velocity and zero acceleration.

General Plane Motion

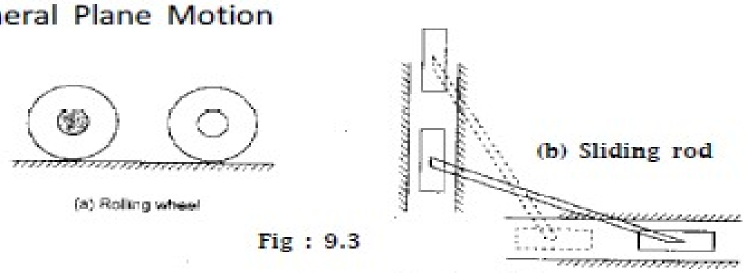


Fig : 9.3

The motion of a rigid body is said to have general plane motion when the body undergoes a combination of translation and rotation. In other words, any plane motion which is neither a rotation nor a translation is referred to as a general plane motion.

Relation between Linear Motion and Angular Motion

Following are the relations between the linear motion and the angular motion.

Initial velocity	u	ω_0
Final velocity	v	ω
Constant acceleration	a	α
Total distance traversed	S	θ
Formula for final velocity	$n = u + at$	$\omega = \omega_0+\alpha t$
Formula for distance covered	$s = ut\frac{1}{2}at^2$	$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
Formula for final velocity	$v^2 = u^2 + 2as$	$\omega^2 = \omega^2_0+2\alpha\theta$
Differential formula for velocity	$v=\frac{ds}{dt}$	$\omega = \frac{d\theta}{dt}$
Differential formula for acceleration	$a =\frac{dv}{dt}$	$a =\frac{d\omega}{dt}=\frac{d^2\theta}{dt^2}$

A body is rotating with an angular velocity of 8 radians/sec. After 5 seconds, the angular velocity of the body becomes 28 rad/sec. Determine the angular acceleration of the body.

Given

$\omega_0 =$ 8 rad/sec ; $\omega =$ 28 rad/sec ; $t=$ 5 sec

Solution

To find the Angular Acceleration of the Body (a) :

We know that $\omega = \omega_0 + \alpha t$

$$28 = 8 + \alpha(5)$$

$$\therefore \alpha = \frac{28-8}{5} = 4 \text{ rad/sec}^2 \qquad \text{(Ans)}$$

A wheel rotating about a fixed axis at 30 r.p.m. is uniformly accelerated for 50 seconds, during which time t makes 40 revolution. Find (i) angular velocity at the end of this interval, and (ii) time required for the speed to reach 80 revolutions per minute.

Given

N_0 = 30 r.p.m. ; t = 50 sec ; θ = 40 revolutions

Solution

We know that,

1 complete revolution = 2π rad

40 complete revolution = $2\pi \times 40 = 80\pi$ rad

$$\text{So, } \theta = 80\pi \text{ rad}$$

$$\omega_0 = \frac{2\pi N_0}{60} = \frac{2\pi \times 30}{60} = \pi \text{ rad/sec}$$

Using the equation,

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$80\pi = \pi \times 50 + \frac{1}{2}\alpha(50)^2$$

$$\therefore \alpha = 0.075 \text{ rad/sec}^2$$

Now using the equation,

$$\omega = \omega_0 + \alpha t$$

$$\omega = \pi + (0.075)\,50$$

$$\omega = \mathbf{6.912 \text{ rad/sec}} \qquad \text{(Ans)}$$

(ii) Time required for the speed to reach 80 r.p.m. (t_1):

When N = 80 r.p.m. ; $\omega = \frac{2\pi(80)}{60} = 8.378$ rad/sec

Then $\omega = \omega_0 + \alpha t_1$.

$$8.378 = \pi + 0.075 \times t_1$$

$$t_1 = \mathbf{69.8 \text{ sec}} \qquad \text{(Ans)}$$

The motion of a disk rotating about a fixed point is given by the relation $\theta = 3\,(1 + e^{-2t})$, where θ is expressed in radians and 't' in seconds. Determine the angular coordinate, velocity and acceleration of the disk when (a) t = 0 and (b) t = 3 seconds.

Given

$$\theta = 3\,(1 + e^{-2t}) \qquad \text{....(i)}$$

Solution

Differentiating equation (i), $$\frac{d\theta}{dt} = \omega = -6e^{-2t} \qquad \text{.....(ii)}$$

Differentiating equation (ii), $$\frac{d\omega}{dt} = \alpha = 12e^{-2t} \qquad \text{.....(iii)}$$

(a) When t = 0

Angular coordinate, $\theta = 3(1+e^{0}) =$ **6 rad** **(Ans)**

Angular velocity, $\omega = -6e^{0} =$ **-6rad/sec** **(Ans)**

and

Angular acceleration, $\alpha = 12\,e^{0} =$ **12 rad/sec^2** **(Ans)**

(b) When t = 3 sec :

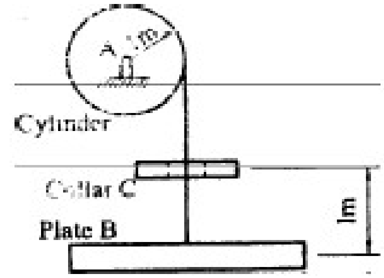
Angular coordinate, $\theta = 3\,(1 + e^{-6}) =$ **3.007 rad.** **(Ans)**

Angular velocity, $\omega = -\,6e^{-6} =$ **- 0.015 rad/sec** **(Ans)**

and

Angular acceleration, $\alpha = 12e^{-6}$ = **0.03 rad/sec^2** **(Ans)**

An inextensible cord going around a homogenous cylinder A of mass 100 kg holds a massless plate B. The collar C of mass 30kg is released from rest in the position shown in fig : and drops upon the plate. Determine the velocity for the collar when it has descended an additional 0.5m after striking the plate. Assume that there is no rebound; that is C and B move downwards locked together and the cord remains taut.



Given

Mass of the cylinder A = 100kg

Mass of the collar C = 30kg

Solution

To find the workdone

The cable is inextensible. Hence the workdone by the internal forces exerted by the cable is zero.

Workdone by the system = 30 x 9.81 x 1.5

= 441.45 J

To find the change in Kinetic Energy

Change in Kinetic Energy due to translatory motionof the weight

$$= \frac{1}{2} \times 30 \times (v_2^2 - v_1^2)$$

$$= 15v_2^2 \qquad (\because v_1 = 0)$$

Change in Kinetic Energy due to rotary motion of the cylinder

$$= \frac{1}{2} I(\omega_2^2 - \omega_1^2)$$

$$= \frac{1}{2}\left(\frac{1}{2}mr^2\right)\omega_2^2 \quad (\because \omega_1 = 0)$$

$$= \frac{1}{2}\left(\frac{1}{2} \times 100 \times 1^2\right) \times \left(\frac{v_2}{1}\right)^2 \quad (\because v_2 = r_2\omega_2)$$

$$= 25\, v_2^2$$

According to work energy principle

Work done by the system = Total change in Kinetic Energy of the system

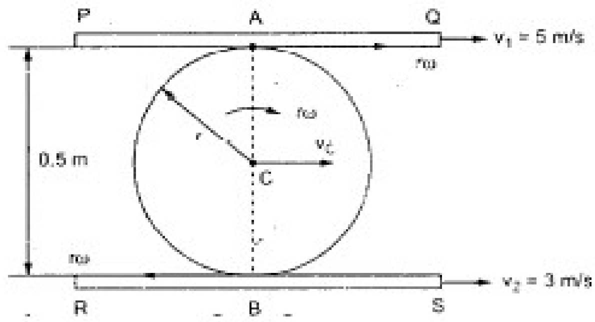
$$441.45 = 15v_2^2 + 25v_2^2$$

$$441.45 = 40v_2^2$$

$$v_2^2 = \frac{441.45}{40} = 11.036$$

$$v_2 = 3.32 \text{ m/s} \qquad \text{(Ans)}$$

A cylindrical roller is in contact at its top and bottom, with two conveyor belts PQ and RS as shown in Fig : If the belts run at the uniform speeds of v_1=5 m/sec and v_2=3 m/sec, find the linear velocity and the angular velocity of the roller. The diameter of the roller may be assumed to be 0.5 m.



Given

v_1 = 5 m/sec;

v_2 = 3 m/sec;

d = 0.5 m

r = 0.25m

Solution

Velocity of the Point A : $\overrightarrow{v_A} = \overrightarrow{v_C} + \overline{v_{A/C}}$

$$5 = v_C + rw \qquad \text{.... (i)}$$

[Put $v_A = v_1$ = 5 m/sec and $\overline{v_{A/C}}$ = rw]

Velocity of Point B ; $\overline{v_B} = \overrightarrow{v_C} + \overline{v_{B/C}}$

$$3 = v_C - rw \qquad \text{.... (ii)}$$

[-ve sign is due to left side direction of velocity $\overrightarrow{v_{B/C}}$]

Adding equations (i) and (ii), we get

$$8 = 2v_C$$

$$\therefore v_C = 4 \text{ m/sec} \qquad \text{(Ans)}$$

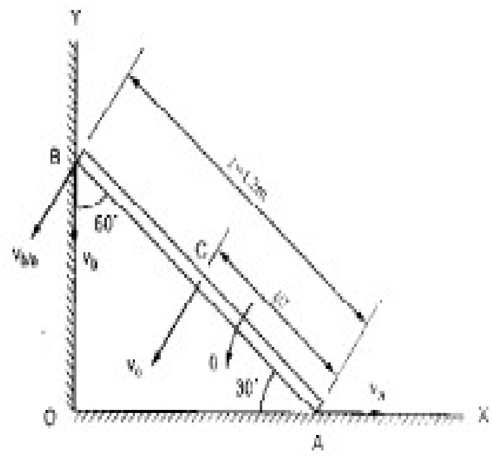
Substituting the value of v_C in equation (i) we get

$$5 = 4+0.25\,w$$

$$\therefore \omega = \frac{5-4}{0.25} = 4 \text{ rad/sec} \qquad \text{(Ans)}$$

A bar of length 1.2m has its ends A and B constrained to move horizontally and vertically as shown in Fig. The end A moves with constant velocity of 6 m/sec horizontally.

Find (a) the angular velocity of the bar, (b) the velocity of the end B, and (c) the velocity of the mid point C of the bar at the instant when the axis of the bar makes an angle of 30^0 with the horizontal axis.



(a)

Given

The arrangement of the given system is shown in fig:

Solution

Velocity of the end B : We know that,

Plane motion = Translation + Rotation about the centre

$$\overline{V_B} = \overline{V_A} + \overline{V_{B/A}} \qquad \text{..... (i)}$$

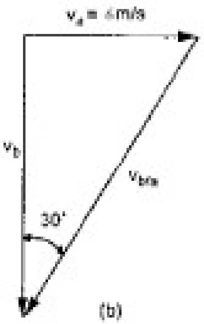
$$\text{But } \overrightarrow{v_{B/A}} = r\omega = 1.2\omega \qquad \text{..... (ii)}$$

The vector diagram of velocities corresponding to the equation (i) can be drawn as shown in fig :

From the vector diagram, $\dfrac{v_A}{v_B} = \tan\theta$

$$\therefore\ v_B = \frac{v_A}{\tan 30^\circ} = \frac{6}{0.5774}$$

$$v_B = 10.39 \text{ m/sec} \qquad \text{(Ans)}$$



(b)

From the vector diagram, $\dfrac{v_A}{v_B} = \tan\theta$

$$\therefore\ v_B = \frac{v_A}{\tan 30^\circ} = \frac{6}{0.5774}$$

$$v_B = \mathbf{10.39\ m/sec} \qquad \textbf{(Ans)}$$

Angular velocity of the bar : From the vector diagram,

$$\frac{v_A}{v_{B/A}} = \sin 30^0 = 0.5$$

$$\therefore\ v_{B/A} = \frac{v_A}{0.5} = \frac{6}{0.5} = 12\ \text{m/sec}$$

But from equation (ii),

$$v_{B/A} = 1.2w$$

$$12 = 1.2w$$

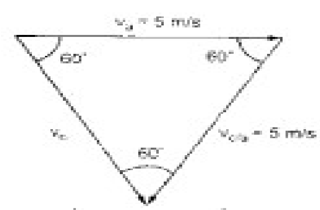
$$\therefore \omega = \frac{12}{1.2} = \mathbf{10\ rad/sec} \qquad \textbf{(Ans)}$$

The velocity of the mid-point C :

$$\overrightarrow{v_C} = \overrightarrow{v_A} + \overrightarrow{v_{C/A}} \qquad \text{[Vector sum]} \qquad \text{.... (iii)}$$

But $\overrightarrow{v_{C/A}} = \dfrac{l}{2} \times \omega = \dfrac{1.2}{2} \times 10 = 6$ m/sec and $v_a = 6$m/sec

The vector diagram of the velocities correspoding to the equation (iii) is drawn as shown in fig. The magnitude of v_C can be determined as.



$$v_C = \sqrt{(v_a)^2 + (v_{C/A})^2 - 2v_A \times v_{C/A}.\cos 60^\circ}$$

$$= \sqrt{(6)^2 + (6)^2 - 2 \times 6 \times \cos 60^\circ}$$

$$v_C = \mathbf{6\ m/sec} \qquad \textbf{(Ans)}$$

UNIT –V

DYNAMICS OF PARTICLES

This chapter is about the vector equation $\vec{F} = m\vec{a}$ for one particle. Concepts and applications include ballistics and planetary motion. The differential equations of motion are set-up in cartesian coordinates and integrated either numerically, or for special simple cases, by hand. Constraints, forces from ropes, rods, chains, floors, rails and guides that can only be found once one knows the acceleration, are not considered.

INTRODUCTION TO DYNAMICS

Dynamics includes:

1. *Kinematics*, which is the study of the geometry of motion. Kinematics is used to relate displacement, velocity, acceleration, and time, without reference to the cause of the motion.
2. *Kinetics*, which is the study of the relation existing between the forces acting on a body, the mass of the body, and the motion of the body. Kinetics is used to predict the motion caused by given forces or to determine the forces required to produce a given motion.

The key tool is, in Newton's words,

"Any change of motion is proportional to the force that acts, and it is made in the direction of the straight line in which that force is acting."

Realizing that the quantification of motion is the product of mass and velocity, and that the rate of change of velocity is acceleration, in modern language we could rephrase Newton's as:

'the net force on a particle is its mass times its acceleration.'

Informally we think 'force causes motion in the direction of the force'. Then, thinking more carefully we fill in the details that in this context 'motion' means acceleration and that the amount of force needed for a given acceleration is also proportional to the mass.

If we define $\vec{F}$ to be the net force on the particle ($\vec{F} = \sum \vec{F}_i$) then linear momentum balance becomes 'Newton's second law',

$$\vec{F} = m\vec{a}.$$

Newton's laws are accurate in a Newtonian reference frame

Acceleration is calculated from position using a particular coordinate system. For our purposes here, a coordinate system is also a reference frame. The calculation of acceleration of a particle depends on how the coordinate system itself is moving. So the simple equation

$$\vec{F} = m\vec{a}$$

has as many different interpretations as there are differently moving coordinate systems (and there are an infinite number of those). In each different coordinate system, the coordinates of a given particle are different from the coordinates in another system. And the calculated accelerations are also different. Sir Isaac Newton was sitting on earth contemplating position relative to the ground at his feet when he noticed that his second law accurately described things like falling apples.

Mechanics is the same on a constant velocity train or plane as on a stationary plane or train. Any reference frame in which Newton's laws are accurate is called a Newtonian reference frame. Sometimes people also call such a frame a Fixed frame, as in 'fixed to the earth' or 'fixed to the stars'. But a Newtonian frame could also be 'fixed' to a constant velocity train or plane. For most engineering purposes a coordinate system attached to the ground under your feet is a good approximation to a Newtonian frame. Fortunately Or else apples would fall differently. Imagine Newton's apple having fallen on some crazy curved path leaving Newton confounded and the subject of mechanics still a mystery. The fall of apples, both in Newton's day and now, is well predicted using Newton's laws and treating the ground as a Newtonian frame. However, if you are interested in trajectory control of satellites, you need to use something more like the ‘fixed stars’ as your (even more accurate) Newtonian reference frame in order to make accurate predictions using Newton’s laws.

MOTION AND ITS TYPES

A body is said to be in motion if it changes its position with respect to its surroundings. The nature of path of displacement of various particles of a body determines the type of motion. The motions may be of the following types:

1. Rectilinear Motion 2. Curvilinear Motion

Rectilinear Motion :

When the particles of a body move in straight parallel path then it is called rectilinear motion.

Curvilinear motion :

When the particles of a body move along a circular arcs (or) curved paths, then it is Curvilinear motion

Instantaneous Velocity :

It is the velocity of particle at any instant of motion. It is the limit of average velocity as the increment of time approaches zero.

Displacement Equations

$v = u + at$

$S = ut + {}^{1}\, at^2$

v2 – u2=2as

An automobile travels 360m in 30 seconds while being accelerated at a constant rate of $0.5 m/s^2$ Find a) its initial velocity b) its final velocity c) distance travelled during first 10 seconds.

Given :

$s = 360m$

$t = 30$ seconds

$a = 0.5\ m/s^2$

Solution :

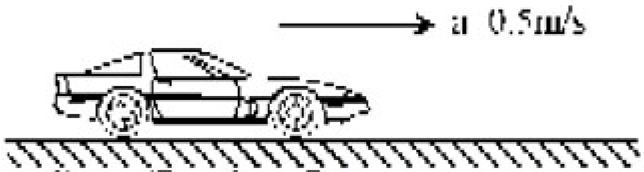


Fig 7.7

a. To find initial velocity (u):

We know,

$$s = ut + \frac{1}{2}at^2$$

a. To Find retardation :

We know,

$$v^2 - u^2 = 2as$$

$$0 - (15)^2 = 2a \times 35$$

$$a = -\frac{225}{70}$$

$$\mathbf{a = -3.21\ m/s^2}$$ **(Ans)**

b. To find time required to stop the car:

$$v = u + at$$

$$0 = 15 + (-3.21) \times t$$

$$3.21\ t = 15$$

$$t = \frac{15}{3.21}$$

t = 4.61 Secconds **(Ans)**

A motorist driving a car at 54km/hr, observes a traffic light 240m ahead turns red. The traffic light is timed to remain red for 24 seconds. If the motorist wishes to pass the light without stopping just as it turns green again, Find

(a) the required uniform deceleration of the car (b) Speed of car as it passes the traffic light.

Given :

Initial velocity, u = 54km/kr

$$= \frac{541000}{3600} \text{ m /sec}$$

$$u = 15 \text{ m/s}$$

Solution :

Consider the journey of train in 3 portions, viz, accelerating, uniform and declereating Refer Fig

Accelerating Journey :

Initial velocity, $u = 0$ $a = 0.25 \text{ m/s}^2$

$t = 1\text{min} = 60$ seconds

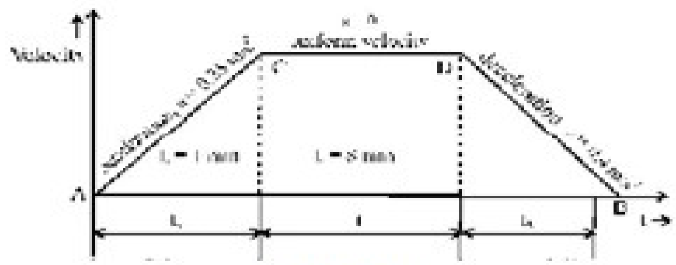


Fig 7.9

We know,

$$S = ut + \frac{1}{2} at^2$$

$$S = 0 + \frac{1}{2} \times 0.25 \times (60)^2$$

$$S = 450 \text{ m}$$

Final velocity $V = u + at$

$= 0 + 0.25 \times$

60 $V = 15$ m/s.

Uniform Velocity Journey:

Initial velocity $u = 15$

m/s $a = 0$

$t = 8$ min

$t = 8 \times 60 = 480$ seconds

We know

$$S = ut + \frac{1}{2}at^2$$

$= 15 \times 480 + 0$ $S = 7200$ m

Distance Travelled by Police Party:

Uniform velocity $u = V = 20$ m/s

$a = 0$

Time = t

We know,

$$S = ut + \frac{1}{2}at^2$$

$$S = 20t + 0$$

$$S = 20t \qquad \text{......(2)}$$

For the police party to overtake the burglar's car, two distance (1) and (2) are same.

$$(t+5)^2 = 20t$$

$$t^2 + 10t + 25 = 20t$$

$$t^2 - 10t + 25 = 20t$$

$$t = \frac{10 \sqrt{100 \quad 100}}{2}$$

t = 5 Seconds **(Ans)**

Striking velocity of a particle dropped from height 'h'

When a particle falls from a certain height 'h' from rest, its initial velocity becomes zero

i.e, u = 0

We know,

$$v^2 - u^2 = 2gh$$

$$v^2 - 0 = 2gh$$

$$v^2 = 2gh$$

$$v = \sqrt{2gh}$$

A body is dropped from rest. find (a) time required for it to acquire a velocity of 6 m/s and (b) time needed to increase its velocity from 16m/s to 23 m/s

Solution :

a. Time required to reach a velocity of 16m/s :

Here,

$u = 0$

$v = 16$ m/s

We know

$v = u + gt$

$16 = 0 + 9.81\,t$

$t = \frac{16}{9.81}$

t = 1.63 Seconds (Ans)

b. Time need to increase its velocity from 16 m/s to 23 m/s:

Here,

$u = 16$m/s

$v = 23$ m/s.

The ball takes some time to reach the ground

Total time the ball remained in air is

T = 2 x 10.09 = 20.18 seconds (Ans)

RESULT

(1) Velocity with which ball was thrown = 99 m/s

(2) Total time the ball remained in air = 20.18 Seconds

A stone dropped into well is heart to strike the water after 3 seconds. Find the depth of well, if velocity of sound is 350 m/s

Given :

Velocity of sound, $v = 350$ m/s

Initial velocity, $u = 0$

Solution :

Let t = time taken by stone to reach bottom of well

Depth of well is

$$h = ut + \frac{1}{2} gt^2$$

$$= 0 + \frac{1}{2} \times 9.81 \times t^2$$

$$h = 4.9 t^2 \quad \ldots\ldots\ldots\ldots (1)$$

We know,

Time taken by sound to reach the top

$$= \frac{\text{Depth of well}}{\text{Velocity of sound}}$$

$$= \frac{h}{350}$$

$$= \frac{4.9\, t^2}{350}$$

By Given:

Total time taken = 3 seconds

Consider 1st particle moving up :

$u_1 = 0$

We know,

$$h_1 = u_1 t + \frac{1}{2} gt^2$$

$$70 = 0 + \frac{1}{2} \times 9.81\, t^2$$

$$70 = 4.9\, t^2$$

$$t = \sqrt{\frac{70}{4.9}} = 3.78 \text{ seconds}$$

Consider the second particle moving down :

$$h_2 = u_2 t - \frac{1}{2} gt^2$$

$$30 = u_2 (3.78) - \frac{1}{2} \times 9.81 \times (3.78)^2$$

$$30 = 3.78\, u_2 - 70$$

$$3.78 u_2 = 100$$

$$u_2 = \frac{100}{3.78}$$

$$\mathbf{u_2 = 26.45 \text{ m/s}} \quad \textbf{(Ans)}$$

The velocity of a moving particle is given by

$v = 13 - t + (0.05)t^3$ where

u = velocity of particle in m/s

t = time in seconds.

The velocity of particle reduces with time. Find the initial velocity and velocity after 5 seconds. Also determine the distance travelled in this time, average velocity and average acceleration.

Given :

$v = 13 - t + (0.05)\ t^3$......... (1)

Solution :

Initial velocity of particle (i.e at t =

0) $v = 13 - 0 + 0.5\ (0)^3$

v = 13 m/s **(Ans)**

Velocity after 5 seconds:

Put t = 5 in equ (1), we get

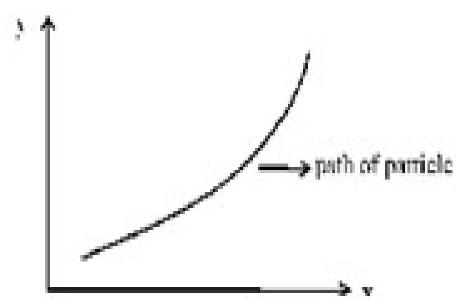
$v = 13 - 5 + (0.05)\ (5)^3$

$= 8 + 6.25$

= 14.25 m/s **(Ans)**

CURVILINEAR MOTION OF PARTICLES

The motion of a particle along the curved path is called as curvilinear motion. If the curved path lies in a single plane, it is termed as plane lies in a single plane, it is termed as plane curvilinear motion. as shown



The motion of a particle is given by the equations

$$x = 2(t+1)^2$$

$$y = \frac{2}{(t \ \ 1)^2}$$

where 'x' and 'y' are expressed in meters and t' in seconds. Find the velocity and acceleration when t = 0.

Solution :

Displacement time relation in x-direction is $x = 2(t+1)^2$

component of velocity in x - direction

$$v_x = \frac{dx}{dt} = 4\ (t+1) \text{ At } t = 0,$$

$v_x = 4\ (0+1) = 4$ m/s component of acceleration in x - direction

$$a_x = \frac{dv_x}{dt}$$

$$a_x = 4 m/s^2$$

Displacement time relation in y - direction is

$$y =$$

$$y = 2\ (t+1)^{-2}$$

Component of velocity in y - direction,

$$v_y = \frac{dy}{dt}$$

$$v_y = -\ 4\ (t+1)^{-3}$$

At t = 0,

$v_y = -4\ (0+1)^{-3}$ $v_y = -4$ m/s

RESULT

At t = 0,

1. Velocity, V = 5.66 m/s

2. acceleration , a = 12.65 m/s^2

The speed of a racing car is increased at a constant rate from 100km/hr to 120 km/hr over a distance of 180m along a curve of 240m radius Determine the magnitude of total acceleration of car after it has travelled 120m along the curve

Given:

Initial velocity, u = 100km/hr = $\frac{100 * 1000}{3600}$

$u = 27.77 \text{ m/s}$

Final velocity, v = 120 km/hr = $\frac{120 * 1000}{3600}$

$v = 33.33 \text{ m/s}$

Normal acceleration, $a_n = \frac{v^2}{r}$

$= \frac{(31.59)^2}{240}$

$a_n = 4.158 \text{ m/s}^2$

Total acceleration, $a = \sqrt{a_t^2 \;\; a_n^2}$

$a = \sqrt{(0.943)^2 \;\; (4.158)^2}$

$\mathbf{a = 4.264 \text{ m/s}^2}$ **(Ans)**

Result

Acceleration, $a = 4.264 \text{ m/s}^2$

What is the smallest radius which should be used for a highway curve if the normal component of the acceleration of a car travelling at 72 km/hr. is not to exceed 0.72 m/s^2?

Given :

Normal acceleration, $a_n = 0.72 \text{m/s}^2$

Velocity, $v = 72 \text{km/hr.}$

$v = \frac{72 \;\; 1000}{3600}$

$v = 20 \text{m/s}$

Solution :

We know,

$a_n = \frac{v^2}{r}$

$r = \frac{v^2}{a_n}$

$$= \frac{(20)^2}{0.72}$$

$= 555.6m$

Smallest radius, **= 555.6 m** **(Ans)**

Total acceleration of particle after 4 seconds is, $a = 144.86\ m/s^2$

PROJECTILE

When a particle is projected upwards at an certain angle to the earth's surface, the particle travels along a curved path. This particle which is thrown into space is called projectile.

For example,

1. A cricket ball thrown into atmosphere
2. A bullet fired from gun
3. A bomb released from moving plane.

Terms used with Projectiles:

Some important terms relating to projectiles are given below. Refer fig 7.30.

a. Projectile Motion:

The motion travelled by the projectile is called as projectile motion.

b. Trajectory:

The path followed by the projectile from the of projection to the point where it meets the ground is called the trajectory of the projectile.

c. Velocity of Projection:

The velocity with which the projectile is thrown into space is called velocity of projection. It is denoted by u.

d. Angle of Projection:

The angle, which the velocity of projection makes with the horizontal or at which a projectile is projected, is called angle of projection. It is denoted by 'T'.

TIME OF FLIGHT, (T)

Let't' be the time taken by particle to reach its maximum height.

Body A is thrown with a velocity of 10m/s at an angle of 60^0 to horizontal. If another body B is thrown at an angle of 45^0 to the horizontal. Find its velocity if it has the same

a. Horizontal Range

b. Maximum Height

c. Time of Flight, as the body A.

Given :

Initial velocity of body 'A' u_A = 10m/s

Angle of projection for 'A', $_A = 60^0$

Angle of projection for body body, 'B', $_B = 45^0$

Solution :

Let intial velocity of body 'B' be u_B **a) If it has same horizontal range:**

$$R_A = R_B$$

$$\frac{u_A^2 \sin 2\ _A}{g} = \frac{u_B^2 \sin 2\ _B}{g}$$

$$(10)^2 \sin 2x60^0 = u_B^2 \sin 2 \times 45^0$$

$$100 \sin 120^0 = u_B^2 \sin 90^0$$

$$100 \times 0.866 = u_B^2 \times 1$$

$$u_B^2 = 86.6$$

$$u_B = 9.3 \text{ m/s}$$ **(Ans)**

) If it has same maximum height:

$$(h_{max})_A = (h_{max})_B$$

$$\frac{\ ^2\ \ ^2}{2g} = \frac{u_B^2 \sin^2\ _B}{2g}$$

horizontal range = 3 × maximum height.

$$\frac{u^2 \sin 2}{g} \cdot \frac{u^2 \sin^2}{2g}$$

$$\sin 2 = \frac{3}{2} \sin^2$$

$$2\sin \cos = \frac{3}{2} \sin^2 \qquad [\ \sin 2 = 2\sin \cos\ ]$$

$$4\cos = 3\sin$$

$$\frac{\sin}{\cos} = \frac{4}{3}$$

$$\tan = 1.33$$

$$= \tan^{-1}(1.33)$$

$$= 53^{o}\ 8'$$ **(Ans)**

A body weighs 50kg on earth. Find its weight (a) on moon where gravitational acceleration is 1.4 m/s2 (b) on the sun, where the gravitational acceleration is 270 m/s2.

Given :

Mass of body, m = 50kg.

acceleration due to gravity in moon, a = 1.4 m/s2

acceleration due to gravity in sun, a = 270 m/s2 s

Solution :

a. Weight body on moon :

We know, weight, W = m × am

W = 50 × 1.4

W = 70 N (Ans)

b. Weight of body on sun :

Weight,

W = m × as

W = 50 × 270

W = 13,500 N (Ans)

First law of Motion (or) Law of Inertia

This law states that, "Every body continues in its state of rest or uniform motion in a straight line, unless it is compelled by same external force to change that state".

Explanation :

The above statement is divided into two parts:

i. If a body is at rest, then so as to set it in motion the external force has to be applied on it.

Example : A book will remain on a table unless it is lifted up by some external force.

ii. If a body is moving with a constant speed along a straight line; then inorder to increase or decrease its speed; a force has to applied in the direction of motion or opposite to the direction of motion.

Example : A ball will move continously with the same speed (provided there is no force of friction. and air resistance) until and unless it is compelled to stop by same external force.

Second Law of Motion

This law states that, " The rate of change of momentum of a body is directly proportional to the applied force and the change takes place in the direction of application of force." This law gives the measure of force and is the fundamental law of dynamics.

Third Law of Motion

This law states that, "To every action, there is an equal and opposite reaction"

DYNAMIC EQUILIBRIUM

The body will be in equilibrium under the action of external force 'F' and the inertia

Force (-ma). This is known as D' Alembert's principle.

So, F = ma is Equation of motion.

A toy train having a mass of 1 kg moves with velocity of 25m/s. If an external force of 10N be applied to the train for a period 0.5 second, find out the final velocity of train when the,

(a) force acts in direction of motion

(b) force acts in opposite direction of motion

Given :

Mass, m = 1kg

Initial velocity, u = 25 m/s

Force, F = 10N

Time, t = 0.5 seconds

Solution :

We know,

$$F = ma$$

$$a = \frac{F}{m}$$

$$= \frac{10}{1}$$

$$a = 10 \text{ m/s}^2$$

a. When Force Acts in Direction of Motion :

When the force acts in direction of motion, the train will have acceleration.

We know,

$$v = u + at$$

$$v = 25 + 10 \times 0.5$$

$$v = 25 + 5$$

Final velocity, $v = 30$ m/s **(Ans)**

We know, v = u + at

v = 25 + (-10) (0.5)

v = 25 – 5

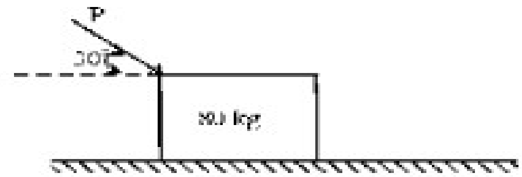
v = 20 m/s

Result

a) Velocity of train when force acts in direction of motion = 30m/s

b) Velocity of train when force acts in opposite direction of motion = 20 m/s

An 80kg block rests on a horizontal plane as shown in fig Find the magnitude of force P required to give the block an acceleration of 2.5 m/s² to the right. The co - efficient of kinetic friction between block and plane is μ = 0.25



Given :

Mass, m = 80 kg

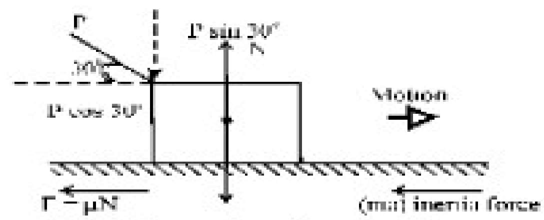
Weight, W = mg = 80 x 9.81 = 785 N

Acceleration, a = 2.5 m/s²

Co-efficient of friction, $\mu = 0.25$

Solution :

The various forces acting on block is shown in fig



Resolving forces vertically,

$\Sigma F_y = 0 \Rightarrow \quad N - W - P \sin 30^0 = 0$

$N - 785 - 0.5\,P = 0$

$N = 785 + 0.5\,P$ ....(1)

Resolving forces horizontally,

$\Sigma F_x = 0$

$P \cos 30^0 - \mu N - ma = 0$

$0.866P - 0.25N - 80 \times 25 = 0$

$0.866\,P = 0.25N + 200$ ....(2)

Substitute value of 'N' in equation (2), we get

$0.866P = 0.25\,(785 + 0.5P) + 200$

$0.866P = 196.25 + 0.125P + 200$

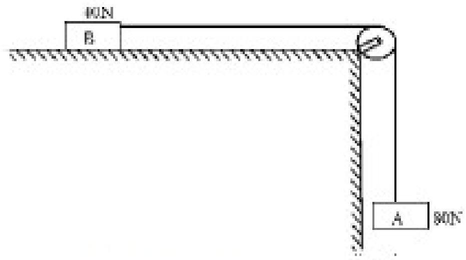
$0.741P = 396.25$

P = 535 N **(Ans)**

RESULT

Magnitude of force, P = 535N

Two blocks A and B of weight 80N and 40 N respectively are connected by inextensible string as shown in fig If the co-efficient of friction between block B and the horizontal plane is $\mu = 0.25$. If the system is released from rest and block A falls through a vertical distance of 2m. What is velocity attained by block A. Take $g = 9.81\ m/s^2$



Given :

$$\text{Weight block A, } W_1 = 80N$$

$$\text{mass of block A, } m_1 = \frac{80}{9.8} = 8.16 \text{ kg}$$

$$\text{weight of block B, } W_2 = 40 \text{ N}$$

$$\text{Mass of block B, } m_2 = \frac{40}{9.8} = 4.08\text{kg}$$

$$\mu = 0.25$$

Distance moved by block A, S = 2m

Initial velocity of block A, u = 0

Solution :

Let 'T' be the tension in the string

Considering block 'A' :

Forces acting on block 'A' is shown in Fig 8.14 (a)

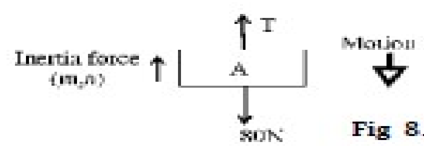


Fig 8.14(a)

Resolving forces vertically,

$$\Sigma F_y = 0 ;$$

$$T + m_1 a - 80 = 0$$

$$T + 8.16\ a = 80 \quad \text{..........(1)}$$

Considering block 'B' :

Forces acting on block 'B' is shown in fig 8.14 (b)

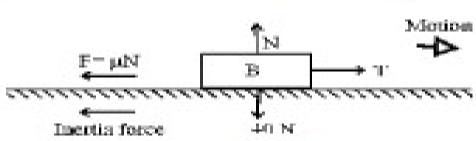


Fig 8.14(b)

Resolving forces vertically,

$$\Sigma F_y = 0 ;$$

$$N - 40 = 0$$

$$N = 40N$$

Resolving horizontally

$$T - \mu N - m_2 a = 0$$
$$T - 0.25 \times 40 - 4.08a = 0$$
$$T - 10 - 4.08a = 0$$
$$T - 4.08a = 10 \qquad \text{........(2)}$$

Subtracting equations (1) & (2) we get,

$$(T + 8.16) - (T - 4.08a) = 80 - 10$$
$$12.24a = 70$$
$$a = 5.72 m/s^2$$

To find velocity of block A :

We know,

$$v^2 - u^2 = 2as$$
$$v^2 - 0 = 2 \times 5.72 \times 2$$
$$v^2 = 22.88$$
$$\mathbf{v = 4.78\ m/s} \qquad \textbf{(Ans)}$$

RESULT

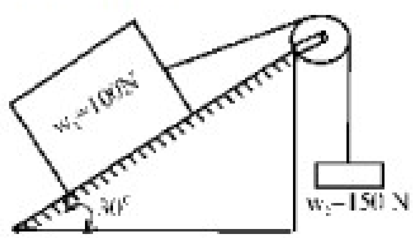
Velocity attained by block A, v = 4.78 m/s

A body weighting 100N rests on a rough inclined plane, as shown in fig It is pulled up the plane, from rest by means of light flexible rope running parallel to the plane. The portion of rope haangs vertically down and carries weight of 150N at the end. Find

(i) Acceleration with which the body moves up the plane

(ii) Tension in the rope

(iii) Distance moved by body in 2seconds, starting from rest. Take $\mu = 0.2$ and $g = 9.8\ m/s^2$



Given :

$$W_1 = 100N, m_1 = \frac{100}{9.81} = 10.19 \text{ kg}$$
$$W_2 = 150N, m_2 = \frac{150}{9.81} = 15.29 \text{ kg}$$
$$\alpha = 30^0$$
$$\mu = 0.2$$
$$t = 0.2$$

t = 2 seconds, Initial velocity, u = 0

Solution :

Let 'T' be the tension in the string

Considering motion of 100N block :

The forces acting on 100 N block is shown in fig 8.16.

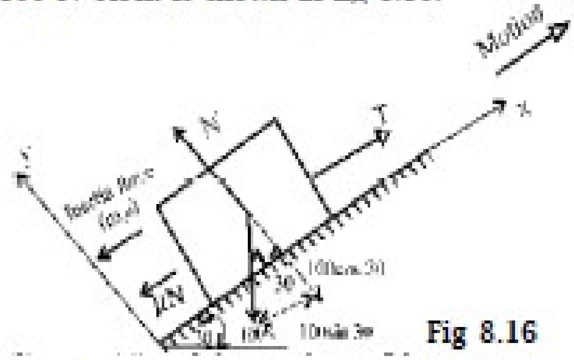


Fig 8.16

Resolving forces perpendicular to plane,

$$\Sigma F_y = 0$$

$$N - 100 \cos 30^0 = 0$$

$$N - 86.6 = 0$$

$$N = 86.6 \text{ N}$$

Resolving forces along th plane,

$$\Sigma F_x = 0,$$

$$T - 100 \sin 30^0 - mN - m_1 a = 0$$

$$T - 100 \sin 30^0 - 0.2 \text{ x } 86.6 - 10.19 \text{ a} = 0$$

$$T - 50 - 17.32 - 10.19a = 0$$

$$T = 67.32 + 10.19a \quad ....(1)$$

Considering motion of 150N block :

The forces acting on 150N block is shown in fig 8.17.

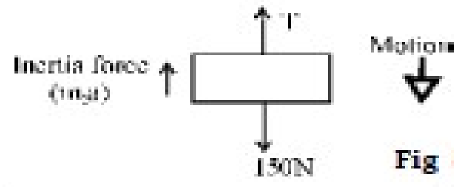


Fig 8.17

Resolving forces vertically,

$$\Sigma F_y = 0;$$

$$T + m_2 a - 150 = 0$$

$$T + 15.29\ a - 150 = 0$$

$$T = 150 - 15.29a \qquad \text{........(2)}$$

Equating, (1) & (2)

$$67.32 + 10.19\ a = 150 - 15.29\ a$$

$$25.48\ a = 82.68$$

acceleration, **$a = 3.25\ m/s^2$** **(Ans)**

To Find Tension in rope:

Substituting the value of 'a' in equation (1),

$$T = 67.32 + 10.19 \times 3.25$$

$$T = 67.32 + 33.06$$

$T = 100.38\ N$ **(Ans)**

To find distance moved by body :

Using relation,

$$S = ut + \frac{1}{2} at^2$$

$$S = 0 + \frac{1}{2} \times 3.25 \times (2)^2$$

$S = 6.5\ m$ **(Ans)**

RESULT

(1) acceleration with which body moves up the plane, $a = 3.25\ m/s^2$

(2) Tension in rope, $T = 100.38N$

(3) Distance moved by the body, $S = 6.5m$

WORK POWER, ENERGY

Concept of Work :

In "Mechancis" work means "accomplishment". A force is said to have done work, if it moves the body, on which it acts, through a certain distance. If a force is not able to produce any displacement, no work is said to have been done.

Work Definition :

Work is defined as the product of force (F) and displacement (s) both being in the same direction.Work is positive or negative according as the force acts in same direction or in the direction opposite to the direction of displacement.

Mathematically,

work done = Force $\times$ distance

$W = F \times S$

POTENTIAL ENERGY (P.E.)

Definition :

The energy which a body possess by virtue of its position or configuration is called potential energy [P.E.]

KINETIC ENERGY (K.E.)

Definition :

The energy which a body possesses by virtue of its motion is known as kinetic energy. It is measured by the amount of work required to be done to bring the body to rest.

Example :

1. Flowing Water 2. Running Car 3. Bullet Fired from Gun 4. Rotating Wheel.

LAW OF CONSERVATION OF ENERGY

Statement :

It states that, "The total amount of energy in the universe is constant, energy can neither be created or destroyed although it may be converted into various forms".

Two blocks of weight 200 N and 100N are connected by a cord passing over a smooth pulley. Find the acceleration of the blocks and the tension in the cord.

Given :

Weight of the two blocks 200N and 100N respectively

Solution :

Let d = distance travelled by the block

workdone by the system = (200-T) $\times$ d+ (100-T) = 100d ....(1)

Change in K.E. of the system

$$= \frac{1}{2}\times\frac{200}{9.81}\times\left(V^2-0\right)+\frac{1}{2}\times\frac{100}{9.81}\left(V^2-0\right)$$

{i.e.,blocks moves opposite sides to each other}

$= 15.29v^2$ ....(2)

According to work energy method

workdone = change in K.E.

$100d = 15.29v^2 \Rightarrow v^2 = 6.54d$

(i) To find the Acceleration

We know

$v^2 = u^2 + 2ad$ ($\because$ initial velocity of the blocks u =0)

$2ad = 6.54 \quad \Rightarrow \quad a = 3.27$ m/sec^2 (Ans)

(ii) To find the Tension in the Cord

Consider 200N block

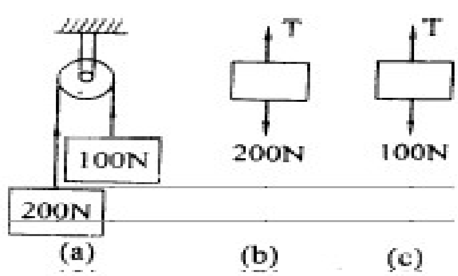


Fig : 8.23

According to work energy method

$$200\text{-}T = \frac{200}{2\times9.81}v^2$$

$$(200\text{-}T)d = \frac{200}{2\times9.81}\times6.54d$$

T = 133.33N (Ans)

A 100N block shown in figure 8.27 is released from rest and slides a distance S down the inclined plane. It strikes the spring which compresses 0.1m, before motion impends up the plane. Take μ=0.25 and spring constant K = 3N/mm. Determine the value of s.

Given :

Weight of the block

$$w = 100N$$

$$\mu = 0.25$$

$$K = 3N/mm = 3 \times 10^3 \ N/m$$

$$\text{Displacement} = (s+0.1)m$$

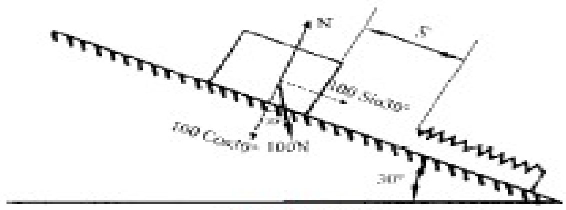


Fig :8.27

Solution :

Resolving forces normal to the plane

$$N\text{-}100\cos 30^0 = 0$$

$$N = 86.6N$$

$$\text{Frictional force} = \mu N$$

$$= 0.25 \times 86.6$$

$$= 21.65N$$

Component of gravitional force along the plane

$$= 100\sin 30^0$$

$$= 50N$$

$$\text{work of the spring} = ½K(0.1)^2$$

$$= ½ \times 3 \times 10^3(0.1)^2$$

$$= 15Nm$$

Now applying the principle of work and energy to the system Σ workdone = Σ Change in Kinetic energy

[(Gravitational Force) – (Frictional Force)]

Displacement – Work of spring = Change in kinetic energy

$$(50\text{-}21.65)\ (S+0.1)\text{-}15 = 0$$

$$28.358+2.835\text{-}15 = 0$$

$$\mathbf{S = 0.429m} \qquad \text{(Ans)}$$

IMPULSE AND MOMENTUM

In the preceeding section, we discussed two methods for solving the problems of motion of the particles. These were based on the application of principle of work and energy. In this section, we shall discuss the third basic method.

Impulsive Force :

The impulsive force is defined as the force which acts for a very short time and yet produces a great change of momentum on the bodies on which it acts.

MOMENTUM

Momentum is defined as the quantity of motion possessed by a body. It is equal to product of mass and velocity of the body.

Momentum = mass x velocity = mV

The unit of momentum is kg.m/s and it is a vector quantity.

PRINCIPLE OF IMPLUSE AND MOMENTUM

Consider a particle of mass 'm' acted upon by a force F. According to Newton's second law,

$$F = ma$$

$$F = m\frac{dV}{dt}$$

$$Fdt = mdV$$

Integrating each side from an initial position at 't_1' when velocity is 'u' to a final position at 't_2' when the velocity is V

$$\int_{t_1}^{t_2} Fdt = \int_{u}^{V} mdV$$

$$\int_{t_1}^{t_2} Fdt = mV - mu$$

This is the expression for principle of impluse and momentum. The integral is known as impluse. Thus by principle of impluse and momentum.

Impluse = Final momentum - Initial momentum

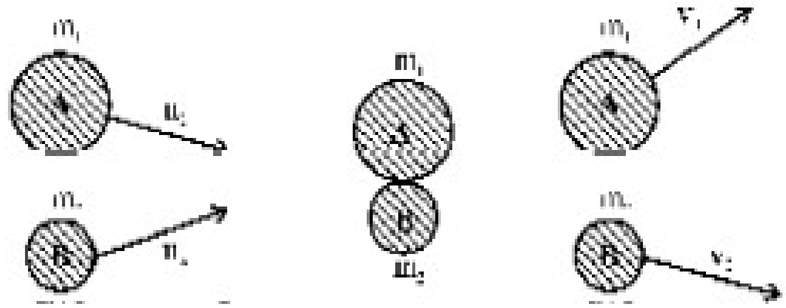
LAW OF CONSERVATION OF MOMENTUM

Statement :

The law of conservation of momentum states that "Total momentum of any group of objects always remains constant, provided if no external forces area acting on them."

Proof :

Consider two bodies A and B moving in same direction with different velocities as shown



Let,

m_1 = mass of the body A

m_2 = mass of the body B

u_1 = Initial velocity of body A

u_2 = Initial velocity of body B

V_1 = Final velocity of body A

V_2 = Final velocity of body B

Before Collision :

Initial momentum of body A = m_1u_1

Initial momentum of body B = m_2u_2

Total momentum of bodies A and B before collision = $m_1u_1+m_2u_2$

After Collision :

Let the bodies A and B collide for a short time 't'

Final momentum of body A = m_1V_1

Final momentum of body B = m_2V_2

Total momentum of bodies A and B aftercollision = $m_1v_1+m_2v_2$

By law of conservation of momentum

Total momentum before collision = Total momentum after collision

$$m_1u_1+m_2u_2 = m_1V_1+m_2V_2$$

A hammer head of mass 1 kg strikes the head of a nail with a velocity of 9 m/s. If the blow lasts 1/80 second, what is the impulse of the blow and average force exerted by the nail on the hammer.

Given :

Mass of the hammer head, m = 1kg

Initial velocity of the hammer, u = o

Final velocity of the hammer, v = 9 m/s

Solution :

$$\text{Time, } t = \frac{1}{80}\,s$$

Impulse of force = Total change in momentum

$$= m\,(v - u) = 1\,(9 - 0)$$

$$= \mathbf{9\ kgm/s} \qquad \textbf{(Ans)}$$

$$\text{and Implusive force, } F = \frac{m(v-u)}{t} = \frac{1(9-0)}{1/80} = 9\times 80$$

$$= \mathbf{720\ N} \qquad \textbf{(Ans)}$$

RESULT

1. Impluse = 9 kg m/s **2. Implusive force = 720N**

A Cricket ball of mass 0.2kg moving with a velocity of 25 m/s is brought to rest by a player in 0.2s. Find the impulse on the ball and the average force applied by the player.

Given :

Initial velocity of the ball, $u = 25$ m/s

Final velocity of the ball, $v = 0$

Time, $t = 0.2$ s

mass of the ball, $m = 0.2$ kg

Solution :

Let F = Average force applied by the player,

Using the relation, $v = u + at$

$$0 = 25 + a \times 0.2$$

$$a = \frac{-25}{0.2} = -125 \text{ m/s}^2$$

(or) Retardation, $a = 125$ m/s^2

$F = ma = 0.2 \times 125 = 25N$ (Ans)

Impulse = $F \times t = 25 \times 0.2 = 5Ns$ (Ans)

COLLISION OF ELASTIC BODIES

When we allow the balls of different materials to fall on a marble floor we find that they rebound to different heights. This property of bodies by virtue of which they rebound, after impact, is called elasticity. The body which rebounds to a greater height is said to be more elastic, than a body which rebounds to a lesser height.

The body which does not rebound at all after the impact is called an inelastic body.

DEFINITIONS :

Restitution :

Whenever the two elastic bodies collide with each other, they tend to compress each other.

Immediately after this they try to regain their original shapes, due to their elasticity. This process of regaining the original shape is called restitution.

Time of Restitution :

The time taken by the two bodies to regain the original shape, after compression is known as time of restitution.

Time of Compression :

The time taken by the two bodies in compression, after the instant of collision, is known as the time of compression.

Time of Collision :

The sum of time of compression and time of restitution is known as the time of collision or period of collision or period of contact.

IMPACT

The phenomenon of collision of two bodies which occurs in a very small interval of time and during which the two bodies exert a very large force and each other is called an impact.

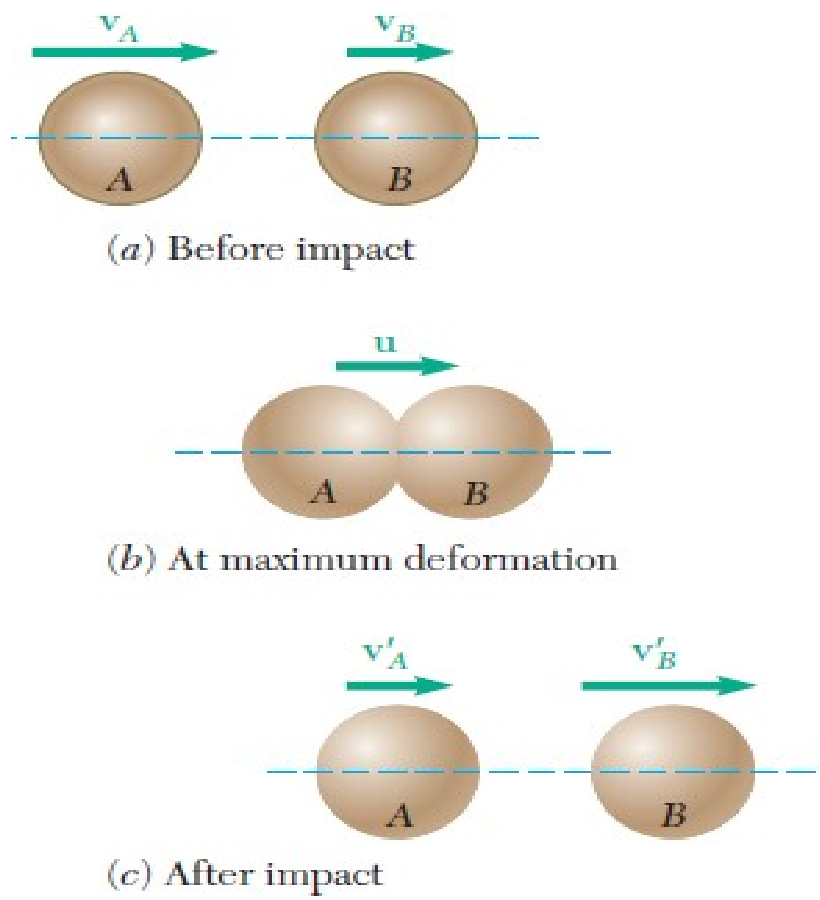
Line of impact :

The common normal to the surfaces of two bodies in contact during the impact, is called the line of impact.

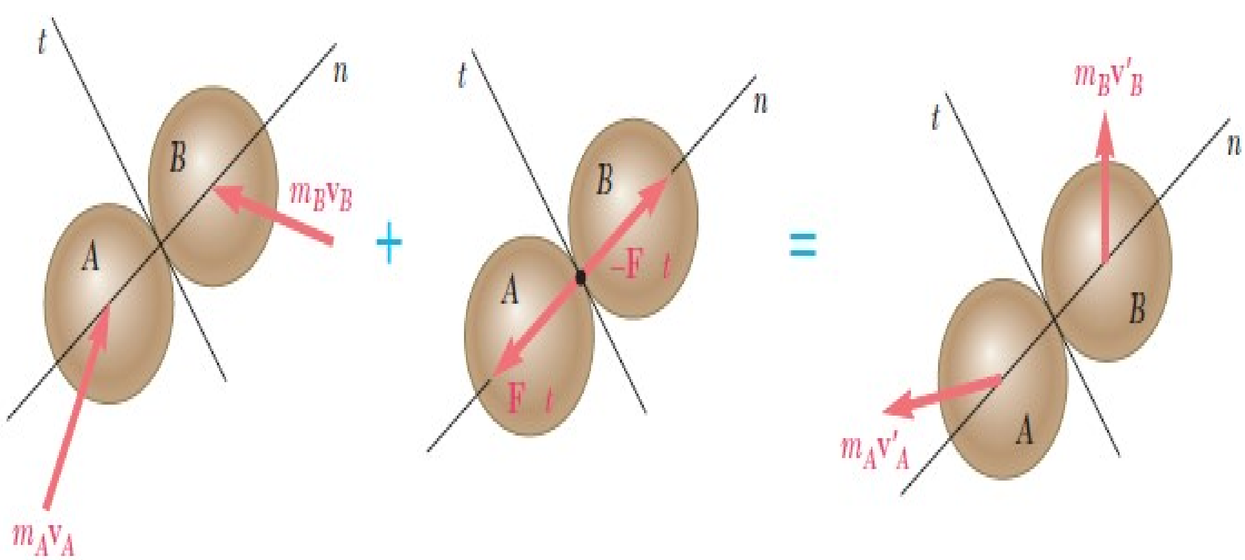
TYPES OF IMPACT

The following are the two types of impacts

(1) Direct Impact (2) Indirect (or) Oblique Impact



Direct impact

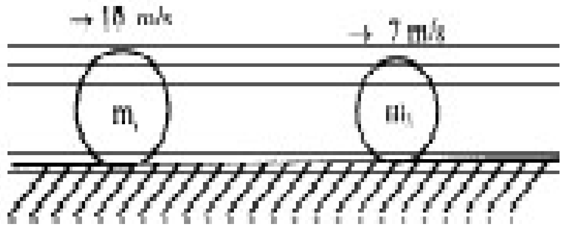


Oblique impact

A body of mass 500 kg is moving with a velocity of 10 m/s strikes another body of mass 300 kg moving at 7 m/s in the same direction. Both the bodies get coupled together due to impact. Find.

(1) The common velocity with which the two bodies will move.

(2) Find the loss of kinetic energy due to impact.



Given :

Mass of first body m_1 = 500kg

Mass of second body m_2 = 300kg

Initial velocity of first body u_1 = 10m/s

Initial velocity of second body u_2 = 7 m/s

Solution :

(i) When the two bodies get coupled, then total mass of the two bodies.

$$M = m_1+m_2$$
$$= 500 + 300$$
$$= 800 \text{ kg}$$

Let

v = Common velocity of the two bodies after impact.

Total momentum before impact.

$$= m_1 u_1+m_2 u_2$$
$$= 500 \times 10 + 300 \times 7$$
$$= 7100 \text{ Kgm/s} \qquad \text{..........(1)}$$

Total momentum after impact

$$= (m_1+m_2)\, v$$
$$= (500+300)v \text{ kgm/s} \qquad \text{..........(2)}$$

According to law of conservation of momentum

Total momentum before impact = Total momentum after impact

$$7100 = (500+300)v$$
$$\mathbf{v = 8.875 \text{ m/s}} \qquad \textbf{(Ans)}$$

ii) Total kinetic energy before impact

$$= ½m_1 u_1+ ½m_2 u_2$$
$$= ½ \times 500 \times 10^2 + ½ \times 300 \times 7^2$$
$$= 25000 + 7350$$
$$= 32350 \text{ Nm}$$

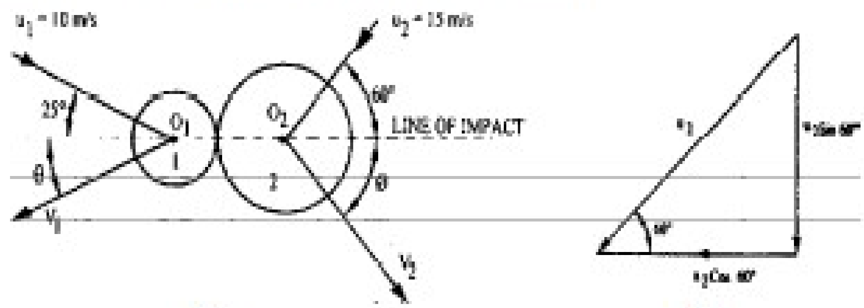
Total kinetic energy after impact

$$= ½ Mv^2$$
$$= ½ \times 800 \times (8.875)^2$$
$$= 31506 \text{ Nm}$$

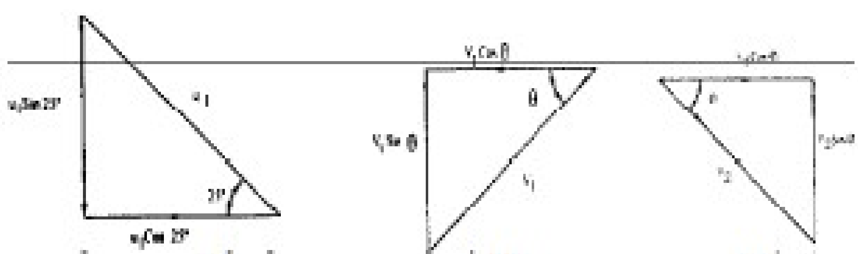
Loss of kinetic energy = Total kinetic energy before impact
Total kinetic energy after impact

$$= 32350-31506$$
$$\mathbf{= 844 \text{ Nm}} \qquad \textbf{(Ans)}$$

A sphere of mass 1 kg moving with a velocity of 10 m/s, strikes on a sphere of mass 1 kg moving with a velocity of 15 m/s. At the instant of impact, the velocities of the balls are inclined at an angle of 25^0 and 60^0 to the line of impact as shown in figure. If coefficient of restitution is 0.9. Calculate (a) the magnitude and direction of first ball velocity after impact and (b) the magnitude and direction of second ball velocity after impact.



(a) (b)



(c) (d) (e)

Given :

Mass of the first ball m_1= 1kg

Initial velocity of the first ball, u_1= 10 m/s

Mass of the second ball, m_2 = 1kg

Initial velocity of the second ball, u_2= 15 m/s

Angle made by the first ball with line of impact before impact $\alpha = 25^0$

Angle made by the second ball with line of impact before impact $\beta = 60^0$

Co-efficient of restitution e = 0.9

Final velocity of the first ball = v_1

Final velocity of the second ball = v_2

Angle made by the first ball with line of impact after impact = θ

Angle made by the second ball with line of impact after impact = ϕ

Solution :

The components of velocity of each ball perpendicular to the line of impact before and after impact is same.

Normal component of initial velocity = Normal component of final velocity

$$u_1 \sin 25^0 = v_1 \sin \theta$$

$$10 \sin 25^0 = v_1 \sin \theta$$

$$v_1 \sin \theta = 4.23 \qquad \text{..............} \ (1)$$

For ball 2

Normal component of initial velocity = Normal component of final velocity

$$u_2 \sin 60^0 = v_2 \sin \phi$$

$$15 \sin 60^0 = v_2 \sin \phi$$

$$12.99 = v_2 \sin \phi \qquad \text{...............}(2)$$

Now according to the law of conservation of momentum

Total momentum along the line of impact before impact =

Total momentum along the line of impact after impact.

$$m_1 u_1 \cos \alpha + m_2 u_2 \cos \beta = m_1 v_1 \cos \theta + m_2 v_2 \cos \phi$$

$$1 \text{ x } 10 \text{ x } \cos 25^0 + 1 \text{ x } 15 \cos 60^0 = 1 \text{ x } v_1 \cos\theta + 1 \text{ x } v_2 \cos\phi$$

$$v_1 \cos\theta + v_2 \cos\phi = 9.06 + 7.5$$

$$= 16.56 \qquad \text{.............} \ (3)$$

The co-efficient of restitution for the indirect impact between two bodies is given by

$$e = \frac{v_2 \cos\phi - v_1 \cos\theta}{u_1 \cos\alpha - u_1 \cos\beta}$$

$$0.9 = \frac{v_2 \text{Cos}\phi - v_1 \text{Cos}\theta}{10 \text{Cos} 25^\circ - 15 \text{Cos} 60^\circ}$$

$$v_2 \cos\phi - v_1 \cos\theta = 0.9 \ (10 \cos 25^0 - 15 \cos 60^0)$$

$$= 0.9 \ (9.06 - 7.5)$$

$$v_2 \cos \theta - v_1 \cos \phi = 1.404 \qquad \text{..............} \ (4)$$

Adding equation (3) and (4), we get

$$2V_2 \cos \phi = 17.964$$

$$v_2 \cos \phi = 8.98 \qquad \text{...............}(5)$$

Substituting the above value in equation (3), we get

$$v_1 \cos\theta + 8.89 = 16.56$$

$$v_1 \cos\theta = 7.578 \qquad \text{.............(6)}$$

Dividing equation (1) by equation (6) we get

$$\frac{v_1 \sin\theta}{v_1 \cos\theta} = \frac{4.23}{7.578}$$

$$\tan\theta = 0.56$$

$$\theta = 29.26^0 \qquad \text{(Ans)}$$

Substituting this value of q in equation (6)

$$v_1 \cos 29.26^0 = 7.578$$

$$v_1 = 8.7 \text{ m/s} \qquad \text{(Ans)}$$

Dividing equation (2) by equation (5)

$$\frac{v_2 \sin\phi}{v_2 \cos\phi} = \frac{12.99}{8.98}$$

$$\tan\phi = 1.44$$

$$\phi = 55.22^0$$

Substituting the value of ϕ in equation (2)

$$v_2 \sin 55.22^0 = 12.99$$

$$\mathbf{v_2 = 15.84 \text{ m/s}} \qquad \text{(Ans)}$$

RESULT

1. Final velocity of first ball v_1 = 8.7 m/s
2. Final velocity of second ball v_2 = 15.84 m/s
3. Direction of first ball $\theta = 29.26^0$
4. Direction of second ball $\phi = 55.22^0$